Dr. Peter U. Georgakis 2003-2004

Acknowledgements

I am extremely lucky and thankful to have a job that I love. I thank my wife Heather, who continues to amaze me with how much support and encouragement she gives me. She is my inspiration, my confidence builder, the person who I always rely on for the soundest advice. For all the things that she does for me big and small, and for always believing in me, I owe the utmost of gratitude. I thank my brother, Tony, who always looked out for his little brother and who flew in from Maryland to share this day with me.

I am grateful to the people instrumental in my hiring at Santa Barbara City College, former President, Dr. Peter MacDougall, former vice-president, Mr. Pat Huglin, and my former dean, Dr. Elaine Cohan, and of course my former chair and office mate, my friend, and truly one of the most special people that I have ever known, Mr. Robert Elmore.

I thank all the faculty and staff for making my first 20 years here quite enjoyable. Finally, I thank my colleagues who have gone before me as faculty lecturer over the last 25 years. I will always remember that our second faculty lecturer, Henry Bagish, and our 18th faculty lecturer, David Lawyer, always shared a warm smile and a kind word and they both left us way too soon.

I thank all my colleagues who provided technical, academic, and administrative support for today's presentation and especially

Jack Ullom, Janet Shapiro, John Kay, Pam Lasker, Barbara Lindemann, Beverly Schwamm, Tom Zeiher, Jim Cheshire, Jim Stevens, Jason Thornell, Joe Thomson, David Wong, Tom Garey, Jason Flynn, Jeff Barnes, Karen Sophia and Grant Lepper, Beverly Schwamm, Kay Bruce, Jeffrey Richardson who spent countless hours working on my visual presentation.

Lecture Dedication

I dedicate my lecture to Caliope Papatsos, former dean of Queens College and long-time mentor who was instrumental in my becoming a teacher; to my wife, Heather, who has been inspiring me for 25 years and always puts me back on track going in the right direction; and to the three people who put joy into my life each day, Alexander, Zoe, and Fiona.

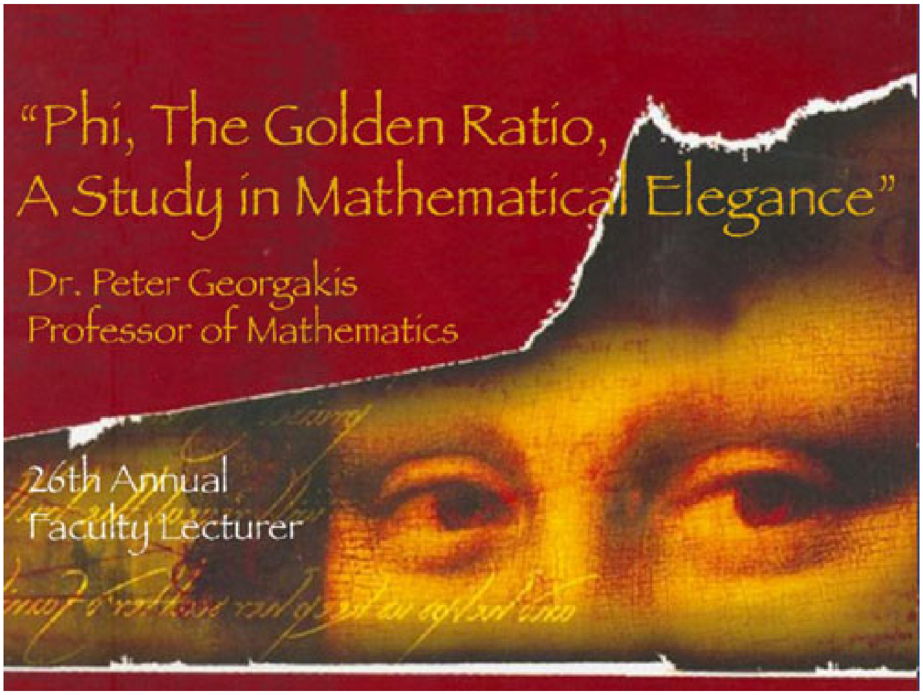
Lecture

Phi, The Golden Ratio

A Study in Mathematical Elegance

Dr. Peter U. Georgakis

Professor, Mathematics



A warm welcome to President Romo, the Board of Trustees, colleagues, students, friends, members of the community, my wife, Heather, and my children, Alexander, Zoe, and Fiona.

I am so flattered to have been chosen to give this talk today. To share the title of faculty lecturer with the former 25 lecturers is an unbelievable honor. I dedicate this lecture to Caliope Papatsos, former dean of Queens College, who was a mentor to me and was instrumental in my becoming a teacher, to my wife, Heather, who has been inspiring me for 25 years, and to the three people who put joy into my life each day, Alexander, Zoe, and Fiona.

Donald Duck Tape:

Donald Duck : [calling out] Hello!

Peter Georgakis : Hello Donald.

Donald Duck : That's me! Where am I?

Peter Georgakis : Santa Barbara City College and we're in Mathmagic Land.

Donald Duck : Mathmagic Land? Never heard of it.

Peter Georgakis : It's where math can be a great adventure.

Donald Duck : Well, who are you?

Peter Georgakis : I'm Peter Georgakis the 26th faculty lecturer and we're going to do some cool stuff today.

Donald Duck : That's for me! What's next?

Peter Georgakis : I'm going to talk about some mathematics.

Donald Duck : Mathematics! That's for eggheads!

Peter Georgakis: Donald it's not for egg-heads, you like to have fun don't you?

Donald Duck : Yeah.

Peter Georgakis : Well, I'm glad you came Donald.

And I am so glad that you all came on this beautiful afternoon, especially to hear a talk on mathematics.

This lecture is about the star of Dan Brown's *"The Da Vinci Code,"* the code itself the Fibonacci series. The Fibonacci series is a number series that Mother Nature is extremely fond of.

The Fibonacci series is found in nature from the petals on flowers to the sunflower. It's found on the human body, the human face, and in the growth patterns of many types of trees and plants. If we take the ratio of any two sequential numbers in this series, we'll find that it falls into an increasingly narrow range generating what is often called the Golden Mean (sometimes called the Golden Proportion,) our number Phi.

For some, the curious mathematical properties of phi are enough to stimulate interest. For others, phi is most interesting due to its surprising presence in architecture, in art, in music and in nature. Phi is an elegant and fascinating number that I know you will enjoy with me.

As W. Somerset Maugham wrote in the first paragraph of his short story, "Mr. Harrington's Washington," "Man has always found it easier to sacrifice his life than to learn the multiplication table."

My first experience with the multiplication table was in elementary school when we were asked to complete among other things multiplying by 9.

$$1 \times 9 = ?$$
$$2 \times 9 = ?$$
$$3 \times 9 = ?$$
$$4 \times 9 = ?$$
$$5 \times 9 = ?$$
$$6 \times 9 = ?$$
$$7 \times 9 = ?$$
$$8 \times 9 = ?$$
$$9 \times 9 = ?$$

I knew that 9 times 1 was 9, but I did not know the rest of them. So I followed the advice of my older brother, Tony, who told me, "Pete, you got to kow what you don't know." Which stil escapes me today, but I tried to aply that principal with the multiplication at hand. I would see how many problems I did not know, 1, 2, 3,... hmmm, 8 of them.

$$1 \times 9 = 9$$
$$2 \times 9 = 1$$
$$3 \times 9 = 2$$
$$4 \times 9 = 3$$
$$5 \times 9 = 4$$
$$6 \times 9 = 5$$
$$7 \times 9 = 6$$
$$8 \times 9 = 7$$
$$9 \times 9 = 8$$

I thought I would check from the bottom up and there were 1, 2, 3, ... 8. Yup, still 8 of them that I did not know.

$$1 \times 9 = 9$$
$$2 \times 9 = 18$$
$$3 \times 9 = 27$$
$$4 \times 9 = 36$$
$$5 \times 9 = 45$$
$$6 \times 9 = 54$$
$$7 \times 9 = 63$$
$$8 \times 9 = 72$$
$$9 \times 9 = 81$$

You can imagine my surprise when Miss Noble, my first grade teacher, told the class the next day that there was a genius in the class and it was me. I look back at that experience every time I hear from my students, "But I got the right answer."

That was just a pattern of numbers and patterns of numbers have always intrigued me. Harvard University Professor Robert Langdon, the hero of Dan Brown's best-selling novel "The Da Vinci Code," is initially baffled by the message, scrawled in invisible ink on the floor of the Louvre in Paris by a dying man with a passion for secret codes.

13 - 3 - 2 - 21 - 1 - 1 - 8 - 5

O, Draconian devil!

Oh, lame saint!

Langdon read the message again and looked up at Fache.

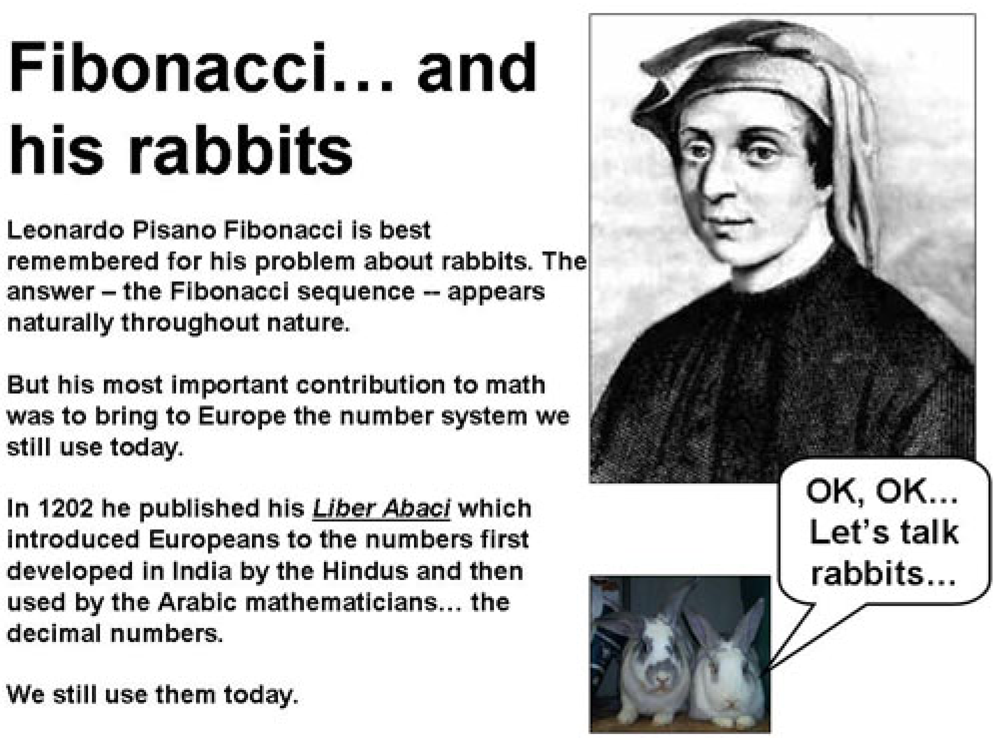
"What the hell does this mean?"

Ah, a comment I have often heard from so many of my math students and the way many of you have felt one time or another sitting in a math class.

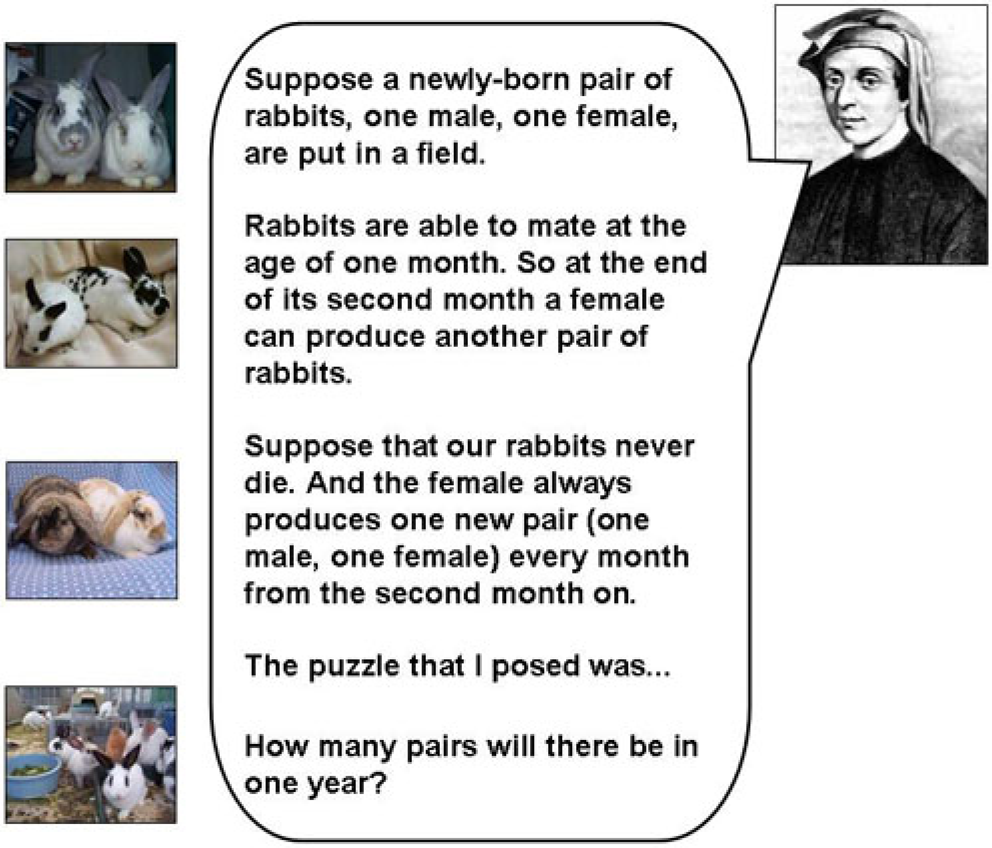
13 - 3 - 2 - 21 - 1 - 1 - 8 - 5

If we rearrange these numbers, we get the following: 1-1-2-3-5-8-13-21 one of the most famous numerical mathematics sequences.

In 1202 AD, Leonardo da Pisa, who was known in Latin as the son of onaccio or "filius Bonaccio, Fibonacii, wrote a book, Liber Abaci (The Book of Calculation). One of the exercises in Liber Abaci was the problem of the rabbits, the most known among the problems he formulated. This one resulted in discovery of the numerical sequence called Fibonacci numbers:



"A man puts a pair of baby rabbits into an enclosed garden. Assuming that each pair of rabbits in the garden bears a new pair every month, which from the second month on itself becomes productive, how many pairs of rabbits will there be in the garden after one year assuming no rabbits die?

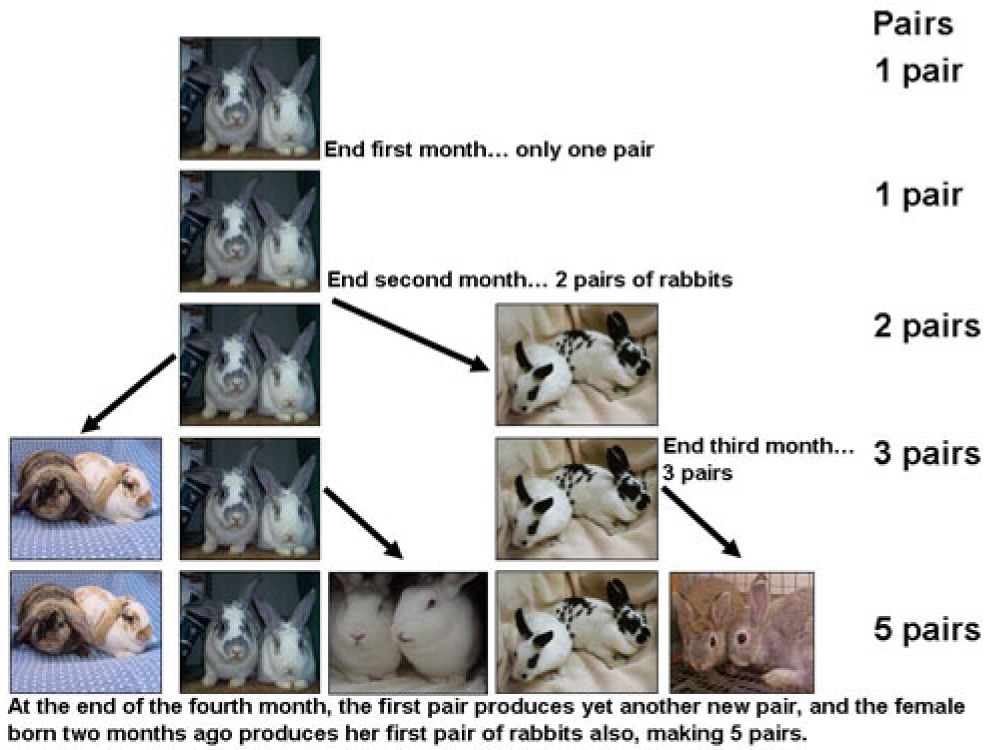


At the end of the first month, they mate, but there is still only 1 pair.

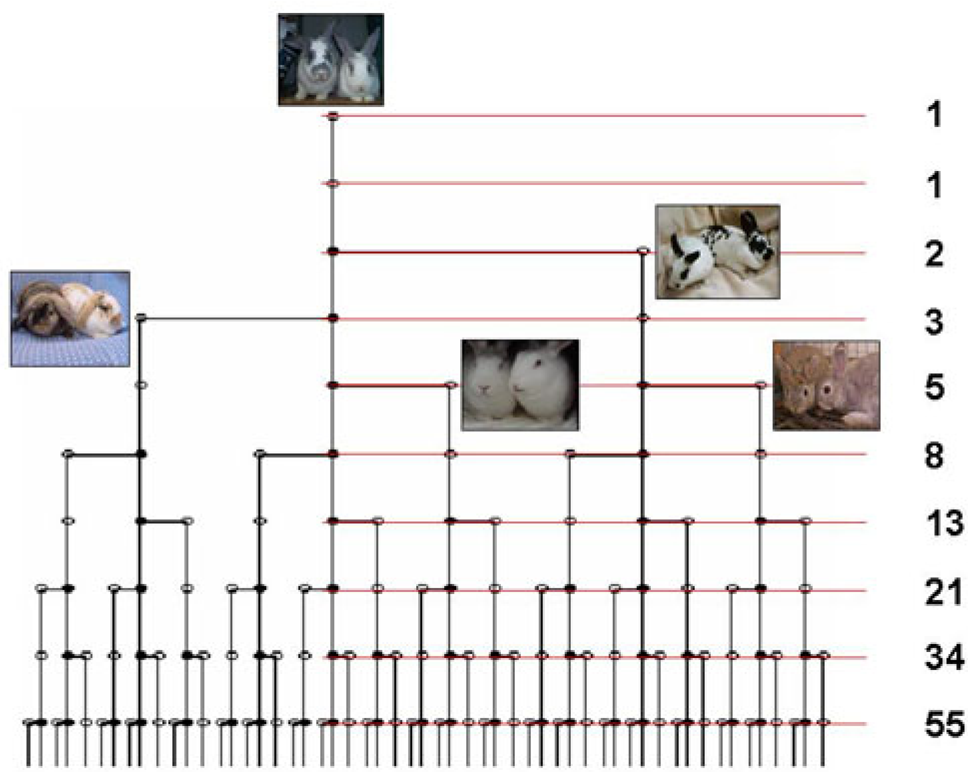
At the end of the second month, the female produces a new pair, so now there are 2 pairs of rabbits in the field.

At the end of the third month, the original female produces a second pair, making 3 pairs in all, in the field.

At the end of the fourth month, the original female has produced yet another new pair, the female born two months ago produces her first pair also, making 5 pairs.



The number of pairs of rabbits in the field at the start of each month is 1, 1, 2, 3, 5, 8, 13, 21, 34.



If we add the previous two to each other, we end up with the third. This is the famous Fibonacci sequence.

$$
\begin{array}{l}
1 \\
1 + 1 = 2 \\
2 + 1 = 3 \\
3 + 2 = 5 \\
5 + 3 = 8 \\
8 + 5 = 13 \\
13 + \ldots\ldots
\end{array}
$$

If you've ever been to a gathering where there are teachers present, you will know they always talk about their school or their students. (Pretty Boring Stuff)

So we will insist that *no two teachers should sit next to each other* along a row of seats and count *how many ways we can seat n people*, if some are teachers T (who cannot be next to each other) and some are not N.

Now I've been to these gatherings and when a teacher shows up you're given a Big T, for Teacher, to wear around your neck.

Everyone else is given an N for Not a Teacher.

If there are no chairs. No one sits down.	1 way
1 chair T or N	2
2 chairs TN, NT, NN	3
3 chairs TNN, NTN, NNT, TNT, NNN	5

There will always be a Fibonacci number of sequences for a given number of chairs, if no two teachers are allowed to sit next to each other!

You can write the sequences using T for Teacher and N for Normal people - oops - I mean Not-a-teacher!!

Let T = Teacher and

N = Not a Teacher

0 chairs:		**1**
1 chair :	**T or N**	**2**
2 chairs:	**TN, NT, or NN**	**3**
3 chairs:	**TNN, NTN, NNT,**	
	TNT, or, NNN	**5**

There's a famous axiom in teaching, the further your office is from the classroom you have to teach in, the more likely you will forget something that you need for that class.

My office is in the IDC building across the way on the third floor. It is almost a certainty that if I am teaching a class on the first floor, I will have forgotten something in my office, usually the over-head graphing calculator. So I race up the stairs. Since I'm in a hurry, I leap up the stairs two at a time - until I get tired and go back to one at a time depending on how many times I have to go back to my office because of the things I have managed to forget.

This puzzle is about what patterns of 1-stair and 2-stair combinations I can make to get to the top of stairs.

For instance, with a single step there is only one possibility, and therefore is only one pattern. Let's write this down as "1" meaning I just step up 1 stair.

For two steps in the staircase, I can take them singly as in "1" and "1" or can leap them in a single two-stair jump, which we'll write as "2". So there are two patterns for two stairs.

1**. for three steps**, I can again take them one at a time.

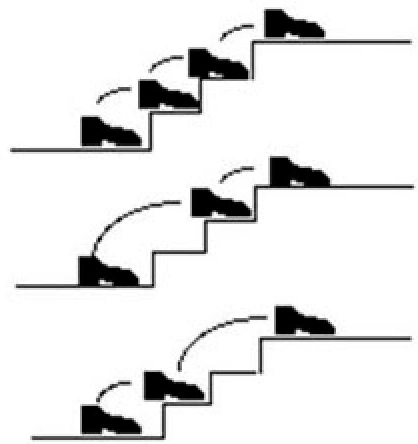
2. I could leap two and then step 1

3. or step 1 and then leap two:

This gives a total of three patterns for three stairs. How many stepping patterns are there for 4 stairs? What about for 5 stairs? or for 6 stairs?

For example, for 3 stairs,
I can go

1: **step-step-step**
or else
2: **leap-step**
or finally
3: **step-leap**



...a total of 3 ways to climb 3 steps.
How many ways are there to climb a set of 4 stairs? 5 stairs? *n* stairs? Why?

These Fibonacci numbers might be merely an Italian mathematical curiosity except for the fact that Mother Nature is extremely fond of this strange sequence of numbers.

Exhibit A: If you count the number of petals in most flowers, you will find that the total is a Fibonacci number. For instance, a white calia lily has one petal, a euphorbia has two,

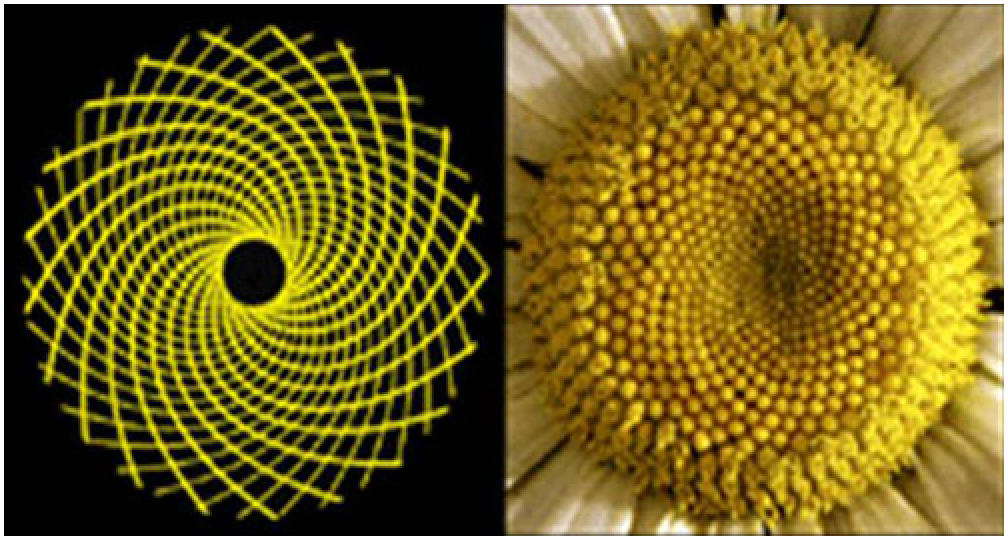
a trillium and an iris have 3 petals,

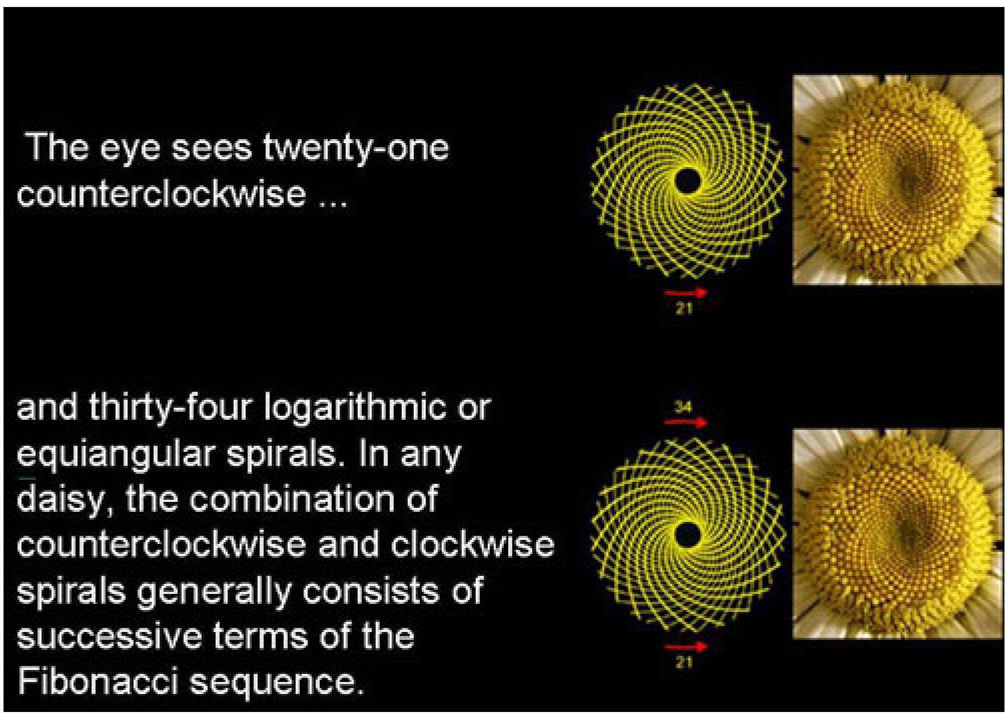
a buttercup 5, a delphinium 8, a ragwort 13, an aster 21,

daisies could have 13, 21, 34, 55 or 89 petals.

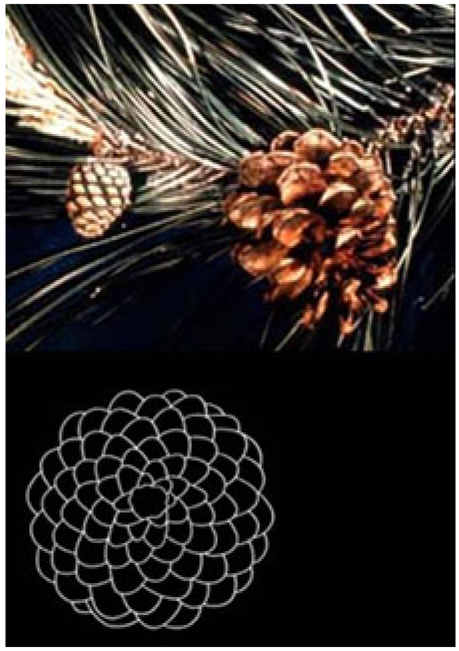
Now if you have a 34 petal daisy, you need to start with she loves me not.

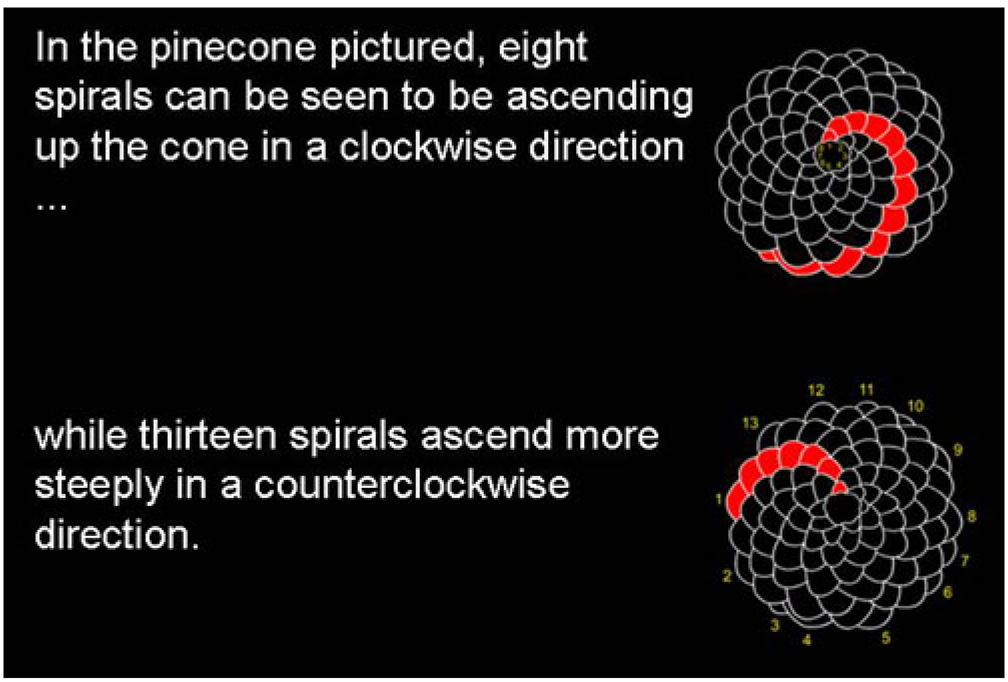
Exhibit B: If you look at a sunflower, you will see a beautiful pattern of two spirals, one running clockwise and the other counterclockwise. If we count the spirals we will find that there are 21 or 34 running clockwise and 34 or 55 running counterclockwise, respectively-all Fibonacci numbers.



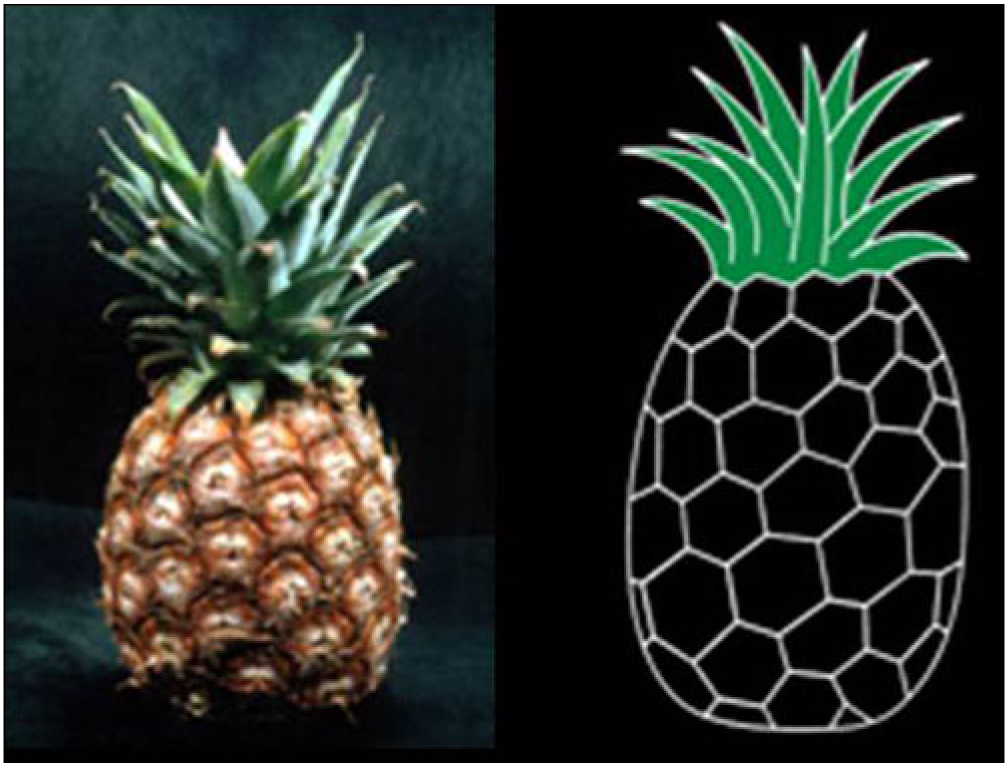


Other flowers exhibit the same phenomenon; the purple coneflower is a good example. Similarly, pinecones often have 8 clockwise spirals and 13 counterclockwise spirals,



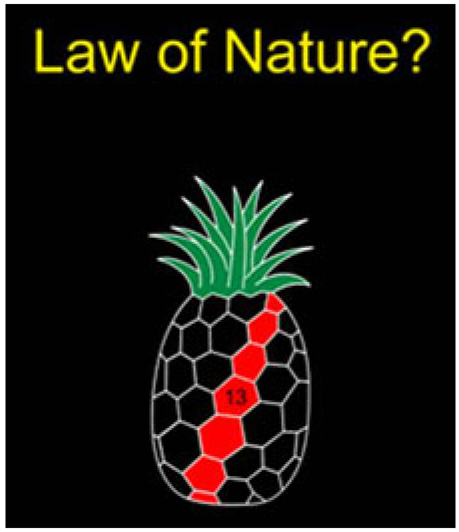


and the pineapple frequently has 5 clockwise spirals and 8 counterclockwise spirals.

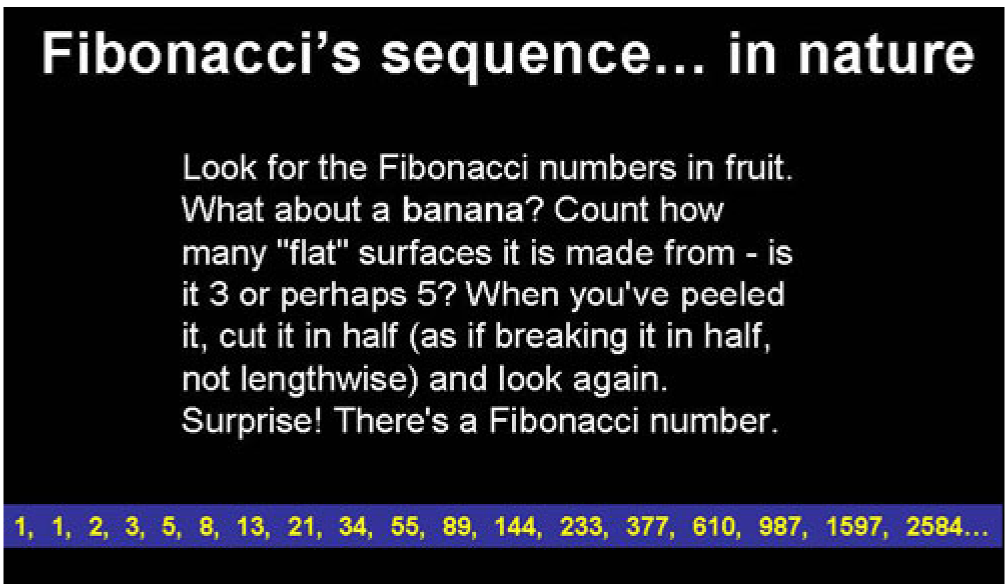


One set of 5 spirals ascends at a shallow angle to the right, ...

a second set of 8 spirals ascends more steeply to the left, ...

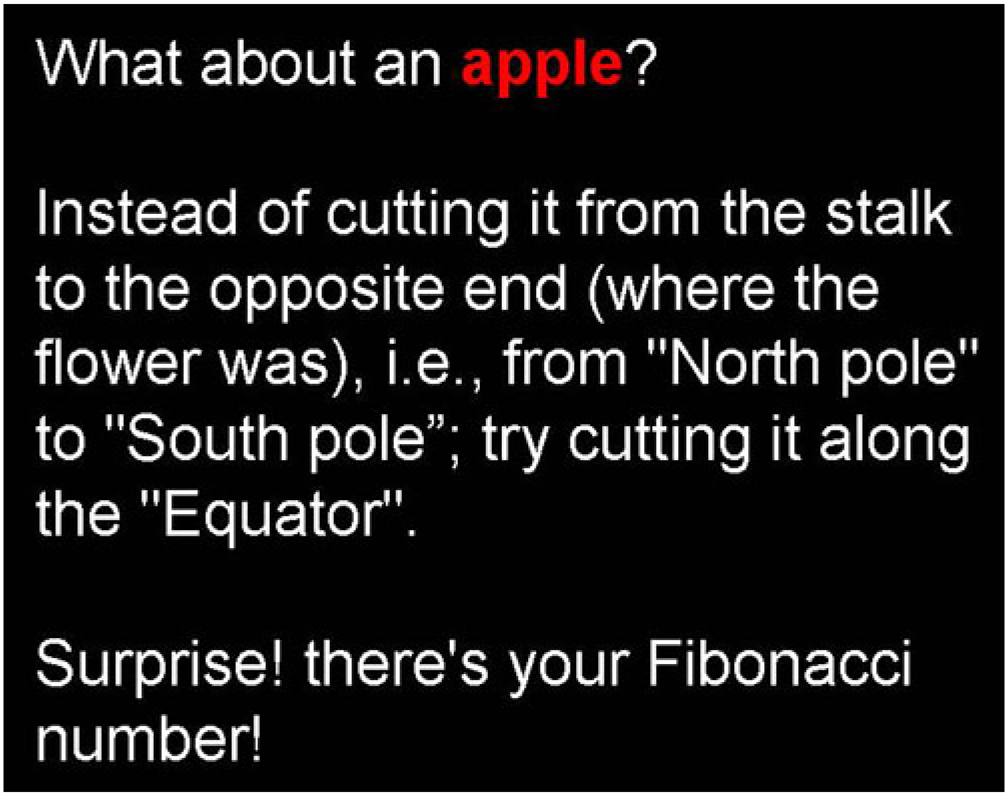


If I take this banana and look at its skin, I can see that it is made up of 5 flat surfaces.

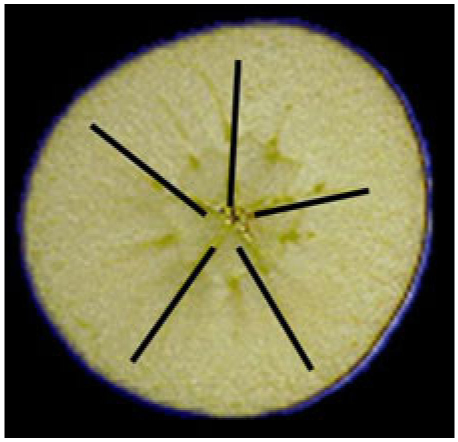


If I cut it in half I find 3 sections.

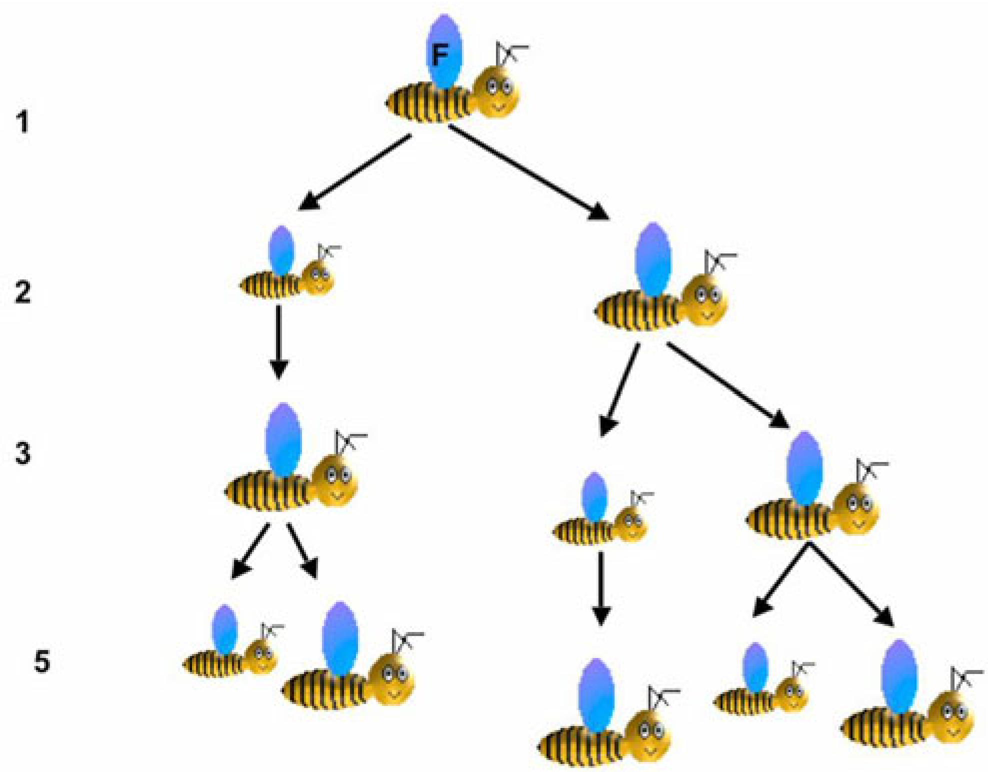
What about this apple? If I cut this apple in half, not North and South, but around the equator



I find 5 sections.



If we look at Honey bees and their Family trees we will see that Queens have 2 parents, a male and a female, while males have 1 parent, a female.



If we take the ratio of any two sequential numbers in this series, we'll find that it falls into an increasingly narrow range?

1/1 = 1

2/1 = 2

3/2 = 1.5

5/3 = 1.6666...

8/5 = 1.6

13/8 = 1.625

21/13 = 1.61538...

34/21 = 1.61904...

and so on, with each addition coming ever closer to some as-yet-undetermined number.

This never-ending, **never-ending** non-repeating, **non-repeating** number Our number, the golden ratio, the golden section, the divine proportion, in many mathematical texts it was referred to as t, tau from the Greek word, tomi which means to cut. At the start of this century, the American mathematician Mark Barr gave this ratio the name of PHI (f), not PHI now I know some of us were in Phi Beta Kappa and Phi Theta Kappa is on this campus, but take it from a good old Greek boy, it's phi the first letter in the name of Phidias the great Greek sculptor who has been credited with many of the sculptures at the Parthenon and made meticulous use of the Golden Ratio in his art work.

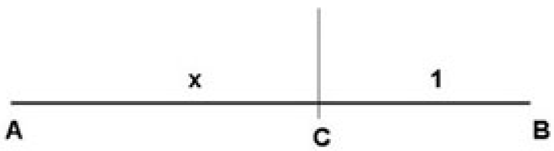
In the Elements, one of the most influential mathematics textbook ever written, Euclid of Alexandria (ca. 300 BC) in book vi propostion 30 states:

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.

Johannes Kepler [1571-1630] said that this was one of the two great treasures of geometry, the other being the Pythagorean Theorem.

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.

So let me see if I can explain this in English. In other words, in the diagram below, point C divides the line in such a way that the ratio of AB to AC is equal to the ratio of AC to CB.



$$\frac{AB}{AC} = \frac{AC}{CB} \quad \text{or} \quad \frac{x+1}{x} = \frac{x}{1}$$

Cross multiply:

$$x^2 = x + 1$$

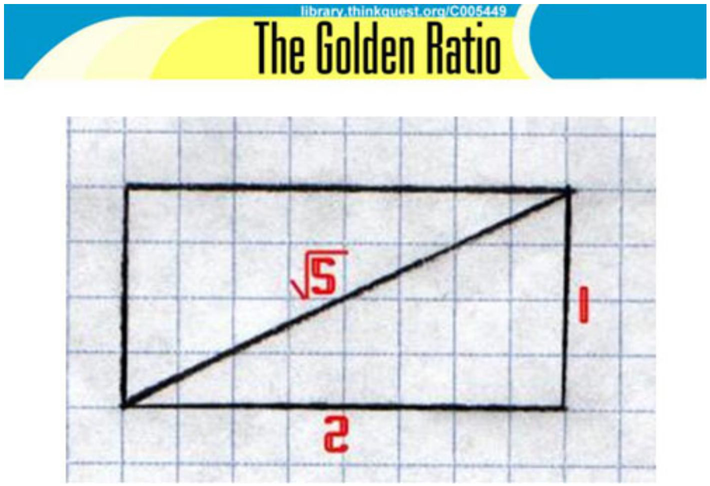
If we cross multiply: $x + 1 = x^2$ and if we do some more algebra here we get $x^2 - x - 1 = 0$ and if we apply the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we get $x = \dfrac{1 \pm \sqrt{5}}{2}$ which shows that in this case the ratio of AB to AC is approximately equal to the irrational number 1.618 (precisely half the sum of 1 and the square root of 5).

$$x^2 = x + 1$$
$$x^2 - x - 1 = 0$$
$$ax^2 + bx + c = 0$$
$$\text{so } a=1,\ b=-1,\ c=-1$$

$$Use \ \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{1+\sqrt{5}}{2} = \phi$$

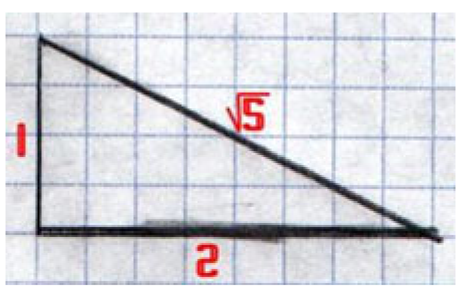
We can come up with this value geometrically in a number of ways. A Golden Rectangle is a rectangle with proportions that are two consecutive numbers from the Fibonacci sequence. Let's use 1 and 2. If we draw a rectangle that has the base of two and the height of one we get the following rectangle.



I've taken the liberty of drawing the diagonal whose length we can find by usingthe Phythagorean theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 1^2 + 2^2$$
$$c^2 = 1 + 4 = 5$$
$$c = \sqrt{5}$$

We can establish that the diagonal is root 5.



The Greeks took this triangle and really cut loose. They moved around the pieces of the triangle to make a super Golden Rectangle:

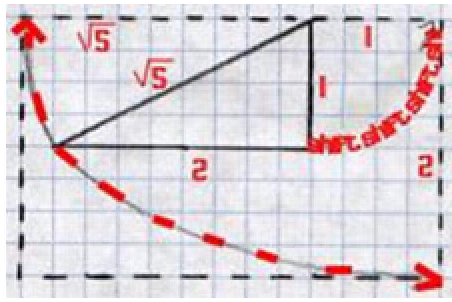
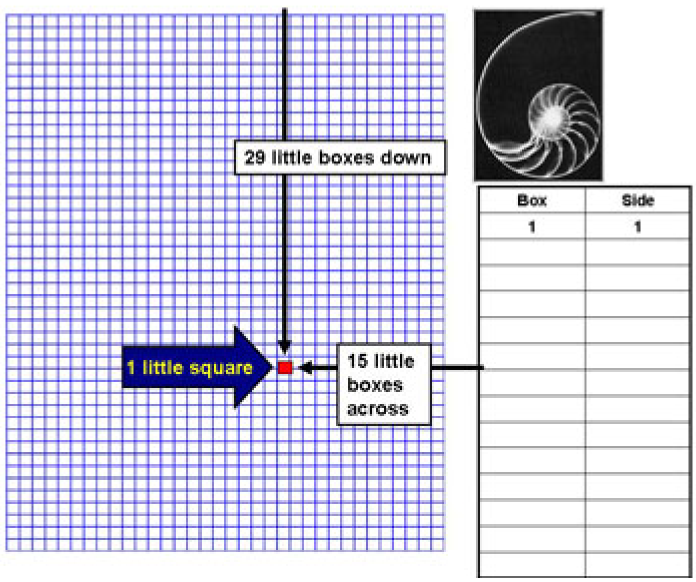
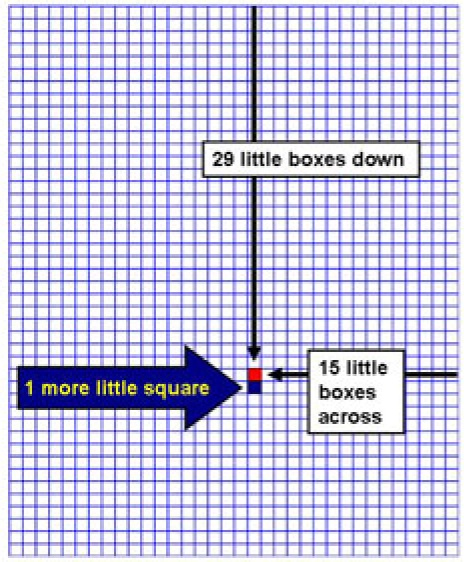
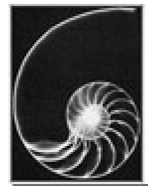


Diagram from ratio of the sides is $\frac{1 \pm \sqrt{5}}{2}$. Sound familiar?

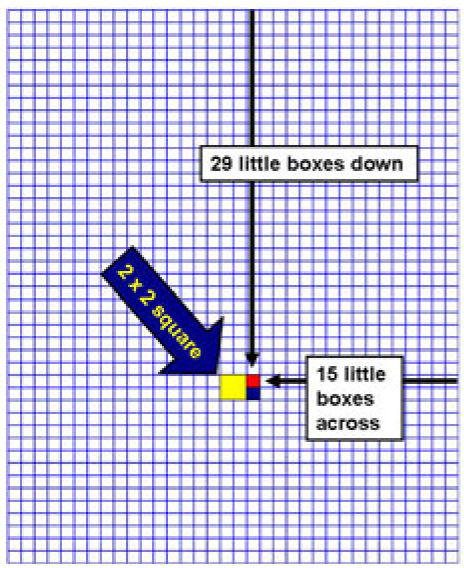
Another way of creating this golden rectangle is by building squares.

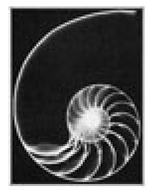




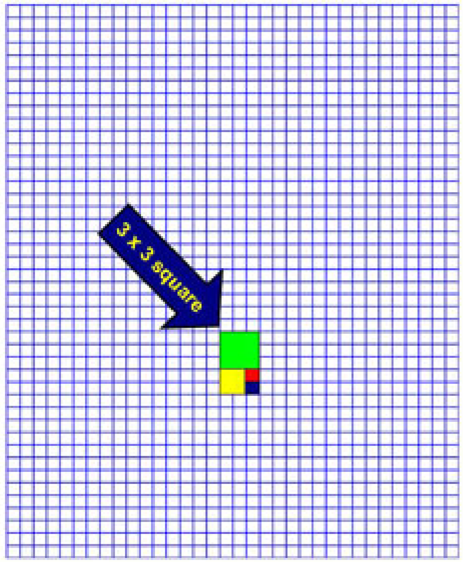


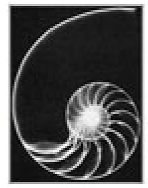
Box	Side
1	1
2	1



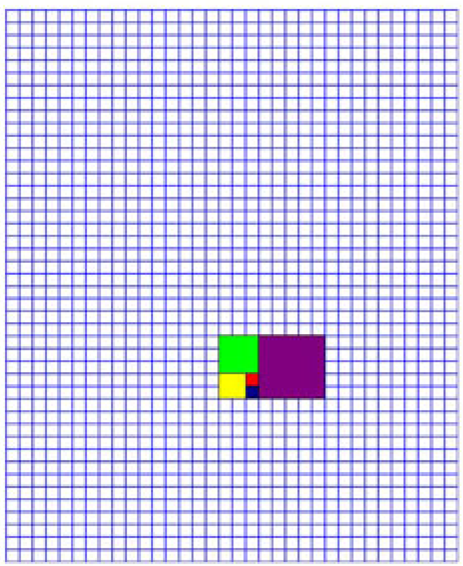


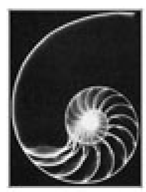
Box	Side
1	1
2	1
3	2



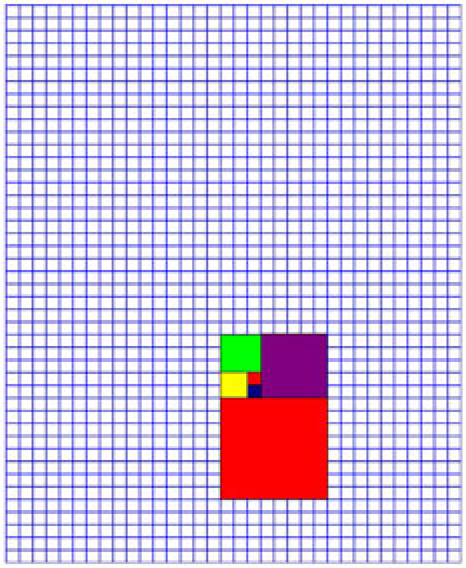


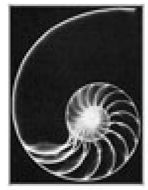
Box	Side
1	1
2	1
3	2
4	3



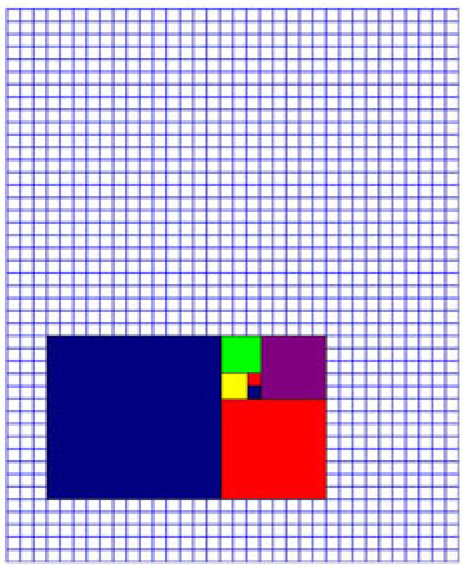


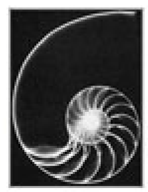
Box	Side
1	1
2	1
3	2
4	3
5	5



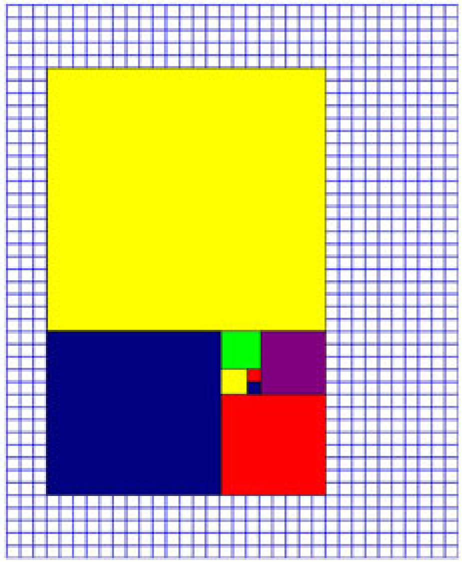


Box	Side
1	1
2	1
3	2
4	3
5	5
6	8





Box	Side
1	1
2	1
3	2
4	3
5	5
6	8
7	13



Box	Side
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21

Film director Robert Zemeckis recently quipped that one of the fundamental commandments of Hollywood filmmaking is to eliminate numbers in a movie for fear that the audience will be confused.

But our number phi is such a fascinating number that I know you will enjoy it with me and not get confused.

Now when we square a number we multiply it by itself. So 3 squared is 9 and 5 squared would be 25. Phi, our golden ratio, is the only number where when you add one to it you end up with its square.

$$\phi^2 = \phi + 1$$

The reciprocal of a number is the number inverted or turned upside down. So 3 is 1/3 and 5 is 1/5. Our number phi, is also the only number that when you subtract one from it you get its inverse.

$$\phi - 1 = \frac{1}{\phi}$$

If you multiply both sides by phi to remove the denominator and then add phi to both sides, we end up with

$$1 + \phi = \phi^2$$

Again, look familiar?

My students always find root equations well, they don't seem to find them appealing.

If we look at this Continuous roots equation

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}}},$$

Remember in Shakespeare's tragedy:

HAMLET: O dear Ophelia, I am ill at these numbers.

AH! We want to scream, but we get a surprising, maybe not so surprising result. If we square both sides of the equation we get

$$x^2 = 1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}}}$$

since by squaring both sides you "undo" one of the square root symbols. Now since

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}}}$$

we can replace our continuous roots of one with x and get

$$x^2 = 1 + x \text{ or } x^2 = x + 1$$

which we know that when solved is our number phi.

The other equation that my students don't like are fractional equations. Let's look at our Continuous frac tions equation:

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}}}$$

:

Whew! So what do you think? Okay you know where we're going here. Since

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}}}$$

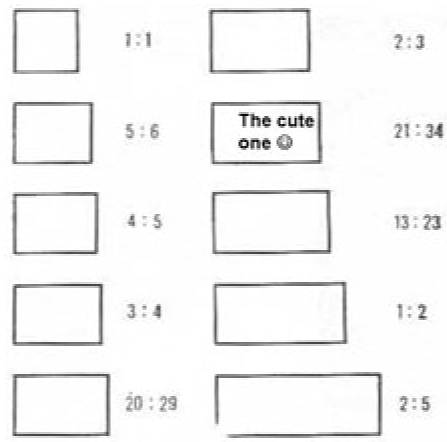
We have x = 1 + 1/x and if we multiply both sides by x we get

$$x^2 = x + 1$$

and if we solved we'd end up with what else our phi our golden ratio.

Mathematicians and artists claim that anything drawn in a golden ratio like the Golden Rectangle is soothing to the eye and visually appealing; no formal study has been conducted to explain this phenomenon, but opinion polls do in fact confirm this aesthetic observation.

The nineteenth century psychologist, Gustav Fechner, reported that, in his research, people identified a rectangle whose length and width had this particular ratio, the Divine Ratio, as the most graceful of all shapes. To evaluate aesthetic feelings 10 white rectangles with the ratio of sides from 1:1 up to 2:5 were presented to participants. They chose the Golden Rectangle, the cute one.

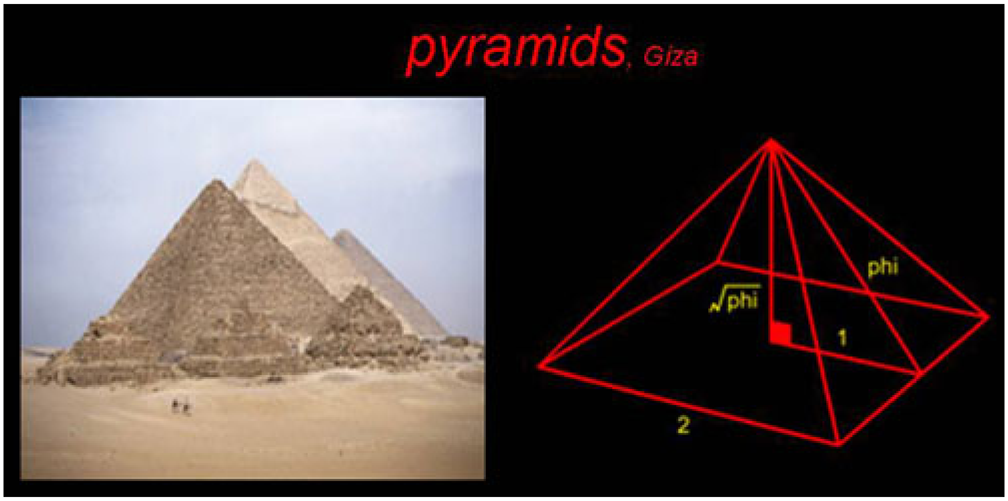


The most obvious example of aesthetic beauty of the golden ratio is right in your wallet. The Golden Rectangle. Your credit card…

For some, its curious mathematical properties are enough to stimulate interest. For others, Phi is most facinating due to its surprising presence in architecture, in art, in music and in nature.

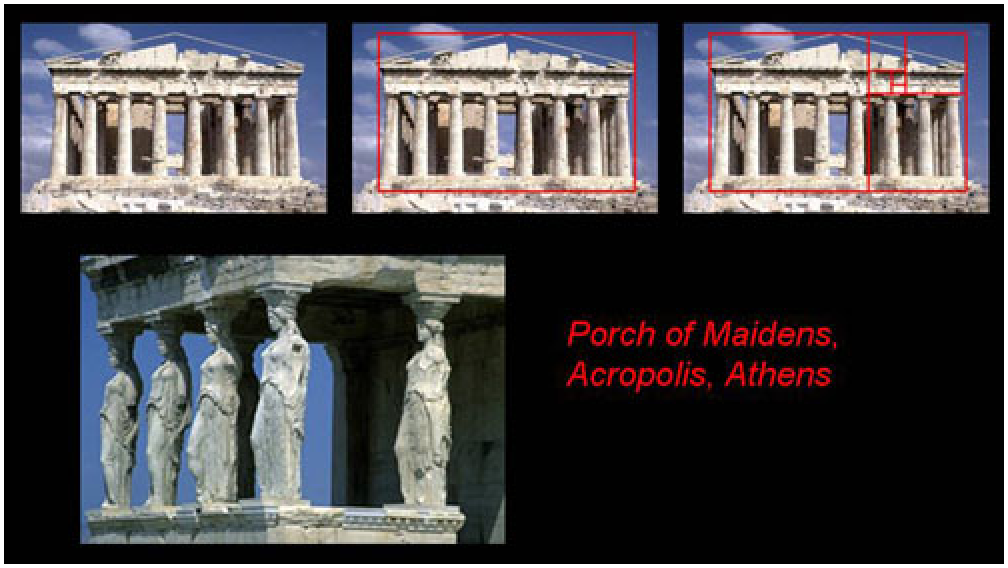
ARCHITECTURE

In architecture there is some contention as to where the Golden Ratio first manifested itself. Some historians claim that it appeared as early as the pyramids at Gayza. Egyptians used the Golden Ratio in creating structures that had a base to height ratio of 1.6, which approximates Phi. The sides of the Great Pyramid rise at an angle of 51 degrees 52 minutes. (You can actually measure this, if you look at a dollar bill. Now if we take the trig function secant of that angle on our calculators we get: Yup, phi.

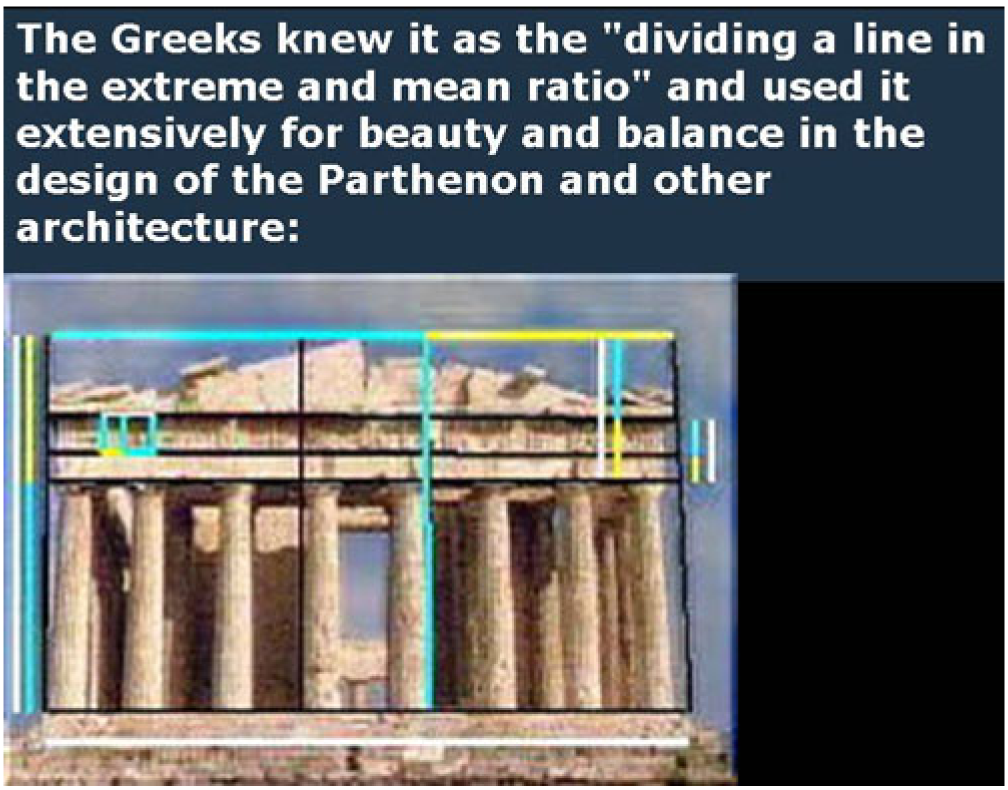


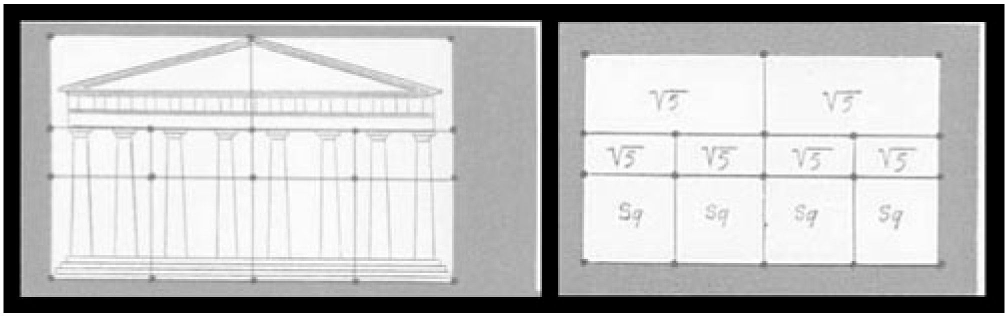
While the intentions of the Egyptians are unknown, we do know that Greeks had begun to calculate far better approximations of Phi and they incorporated it into the construction of the Parthenon, one of their greatest architectural achievements.

The Greek sculptor Phidias sculptured many items including the bands of sculpture that run above the columns of the Parthenon extensively using the golden ration in his work.

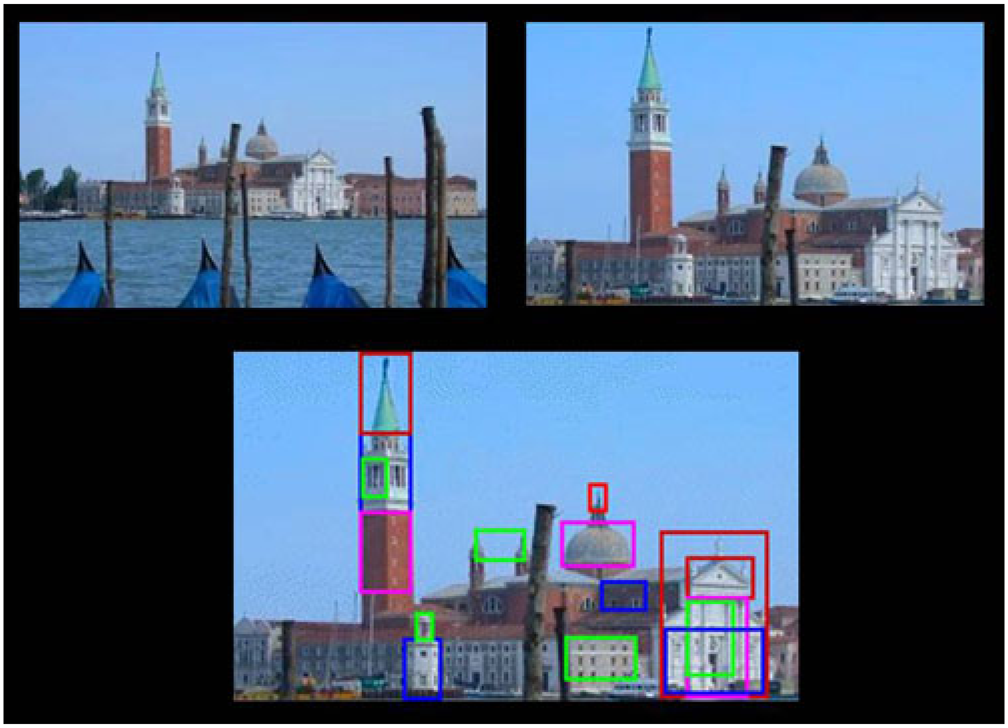


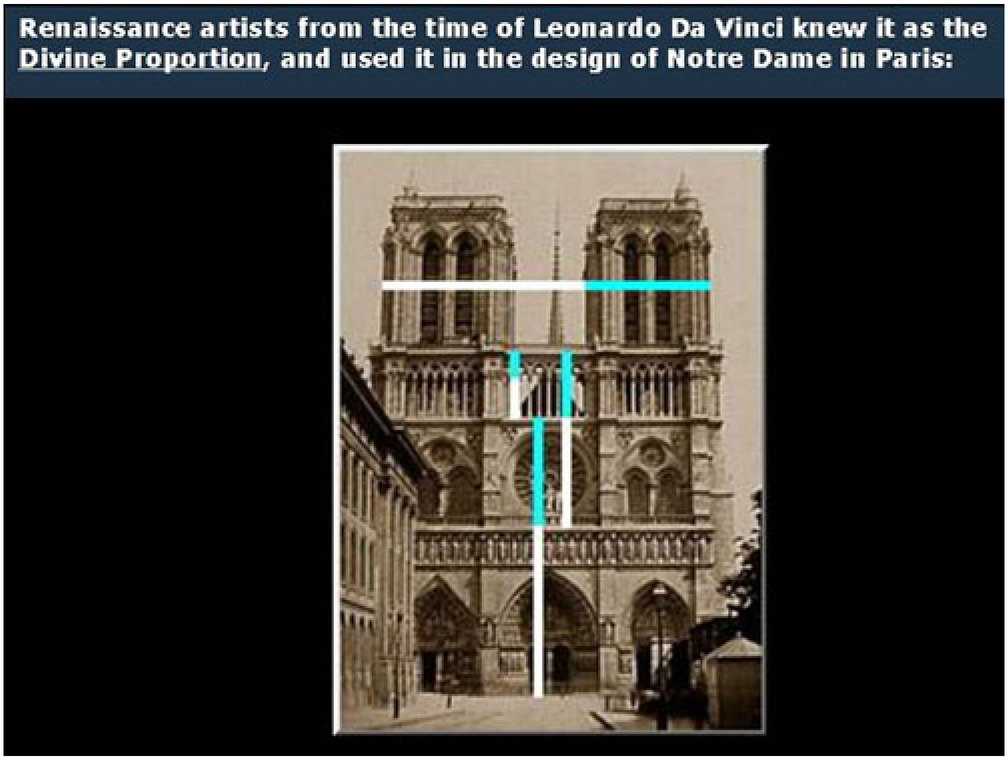
The proportions of the building itself form a Golden Rectangle and the facade of the Parthenon was designed around the proportions of two large and four small Golden Rectangles.





Renaissance artists used it in the design of Notre Dame as well as the Basilica of San Giorgio Maggiore in Venice.

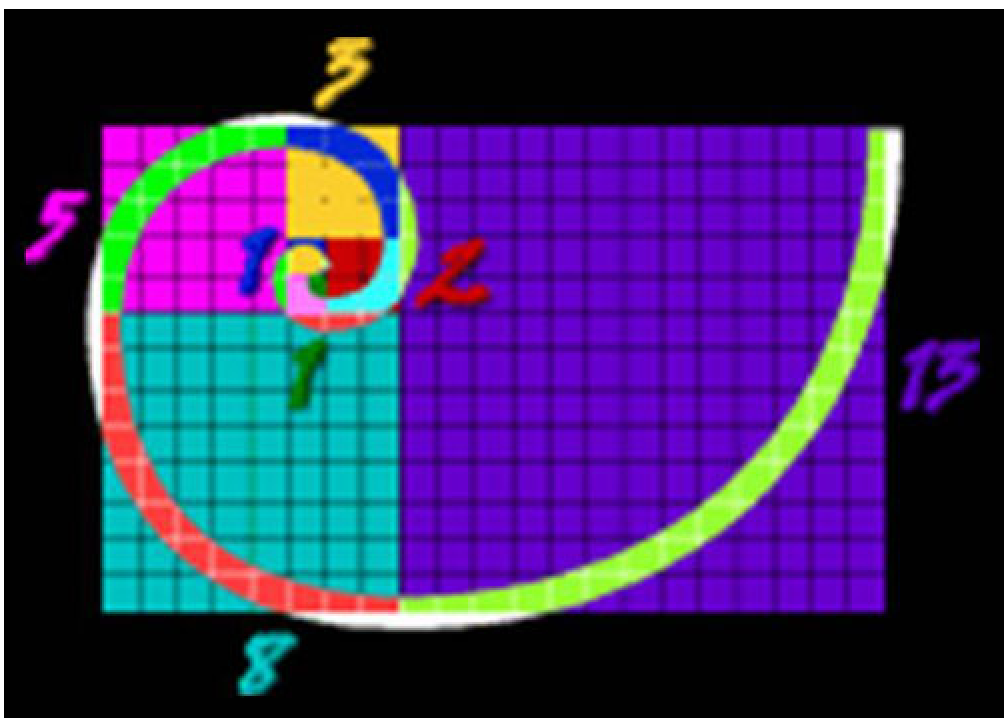


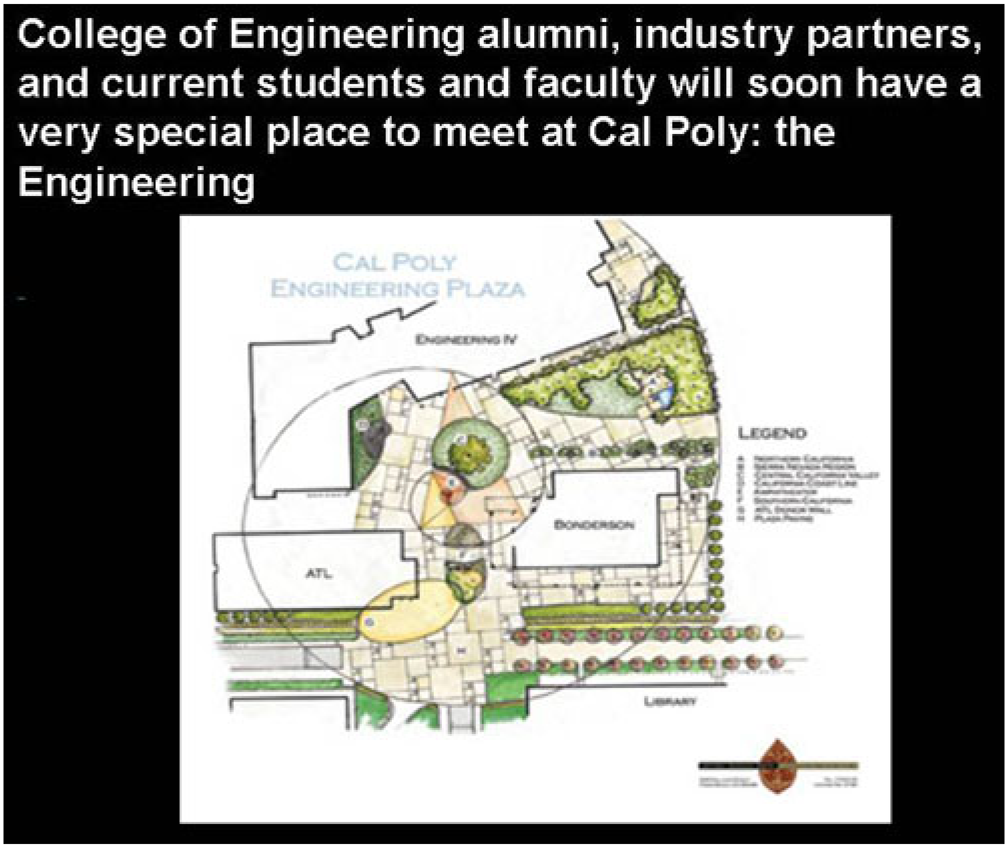


Charles -Edouard Jeanneret (later called Le Corbooseeay) made many trips to Notre Dame. Notre Dame fascinated him especially the way in which they used the Golden ratio in it. He also went to Athens where he spent months studying the Partenon and other ancient Greek buildings.

In fact, he said, "The Parthenon is certainly one of the purest works of art that man ever made. He was amazed at the way that the Greeks used the Golden ratio throughout their work which seemed so inspirational to him. Le Corbooseeay spent much of his life showing the world this great ratio and used golden rectangles to build windows in his design of the United Nations building in New York City.

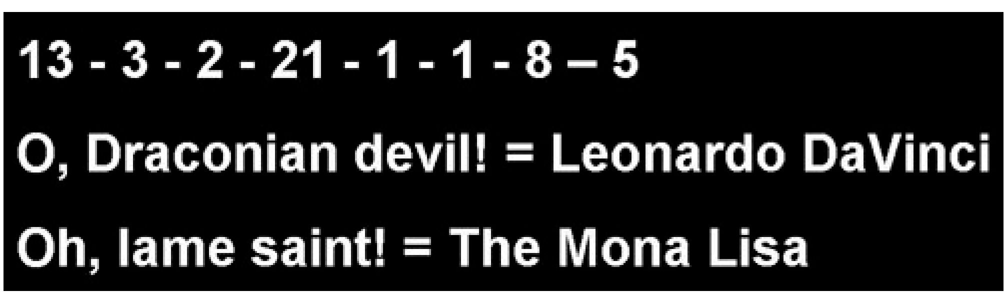
Most recently and just 90 miles up the road. The Cal Poly engineering plaza has been planned for late 2005 and the landscaping is in the shape of a golden spiral or nautilus based on the Fibonacci sequence.



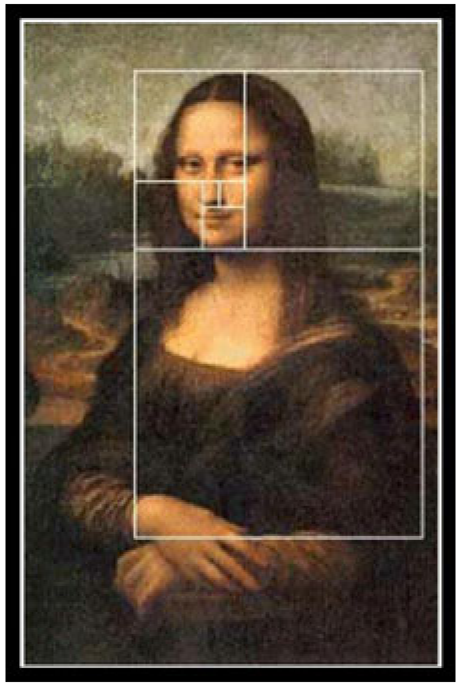


ART

As with architecture, the Golden Ratio manifests itself in art. Remember our Harvard University Professor Robert Langdon, from The Da Vinci Code:

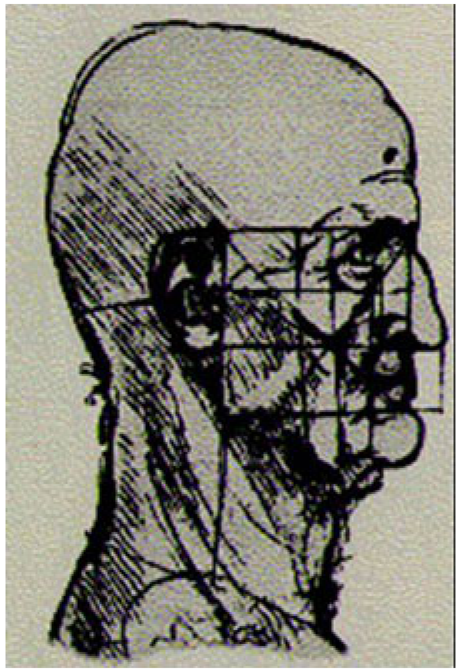


Leoardo da Vinci's artwork was influenced by the golden mean, brought on by the writings of Luca Pacioli in his book *Divina pro* ***por*** *tsionay* (On Divine Proportion). It is striking that the dimensional ratios of the Mona Lisa, perhaps the most recognizable painting in the entire world, are in fact the Golden Ratio. The actual paintig is a golden rectangle. A Golden Rectangle is apparent in the shape of the subject's head. This rectangle can further be broken down into smaller Golden Rectangles. Specifically, if we split the rectangle around her head with a line drawn where her eyes are, we find another. Da Vinci was so intrigued by Phi and its visual appeal that it influenced his developement of perspective in art.

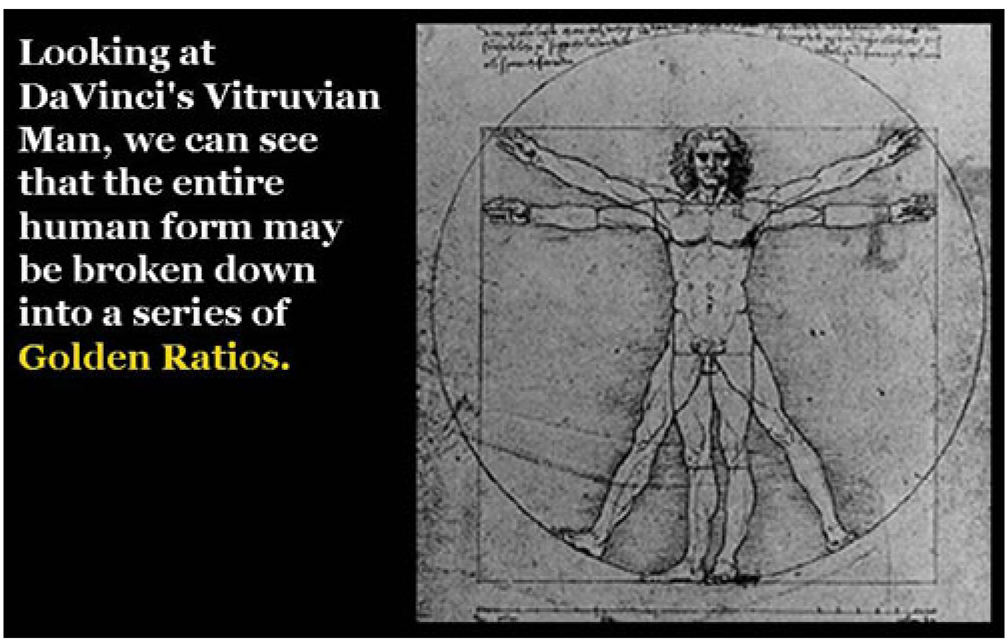


The golden section is present in his unfinished work St. Jerome which fits into a golden rectangle.

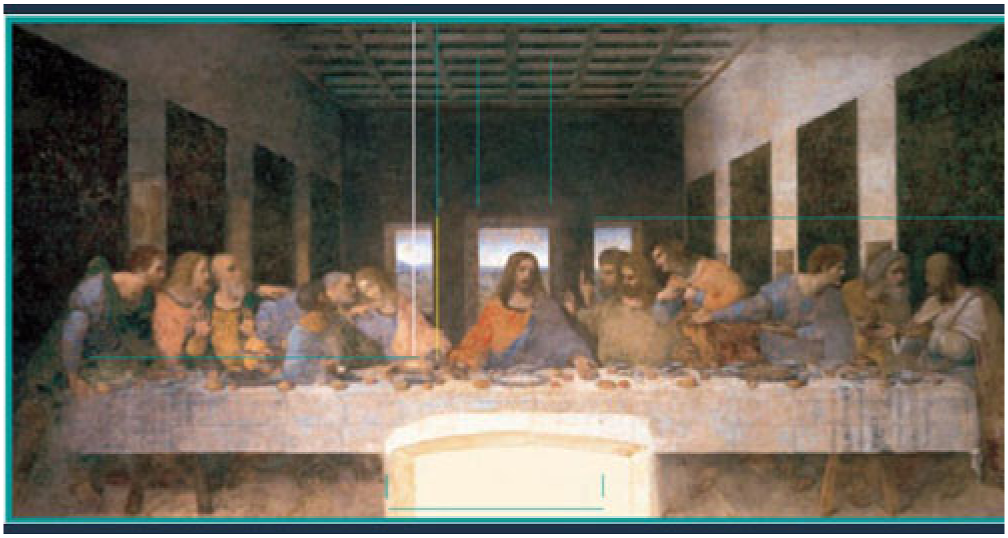
One can see the symmetry in a face Da Vinci's drawing of an old man. The artist overlaid the picture with a square subdivided into rectangles, golden rectangles.



His Vitruvian Man illustrates the use of the golden section extensively.



The painting *The Last Supper* clearly show use of the golden mena.

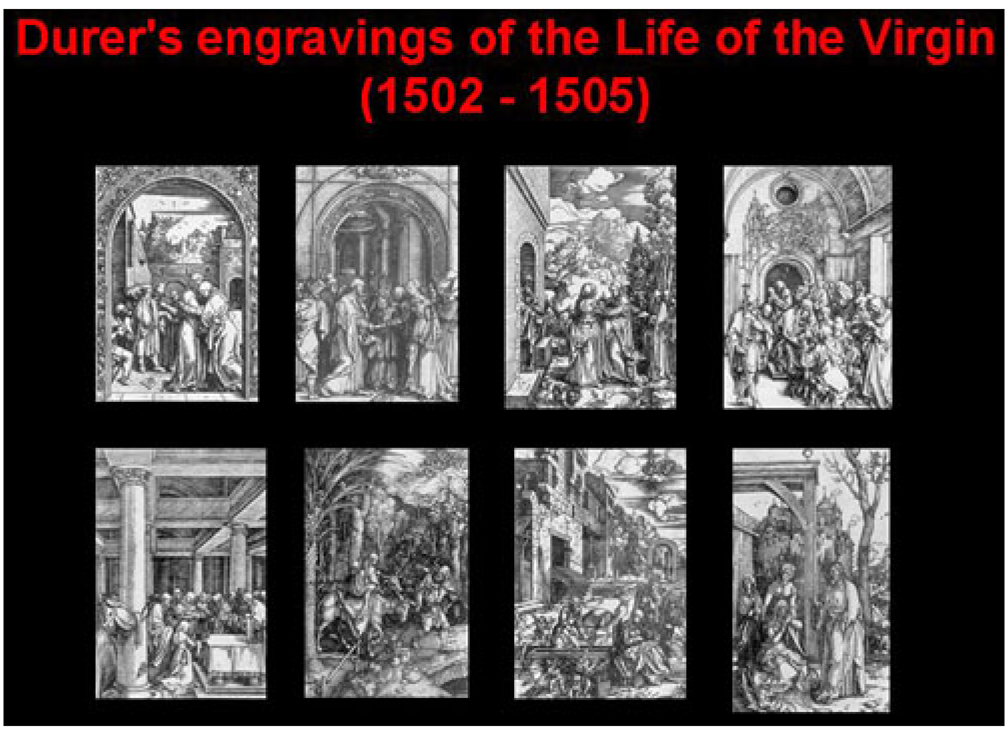


Many art textbooks suggest placing objects not in the center of a picture but on the sides abiding by the law of the golden mean and making the picture more enjoyable to observe Da Vinci's theories of perspective are still largely influential today, thus Phi can be found in painting and photography, whether the artist knowingly incorporates it or not.

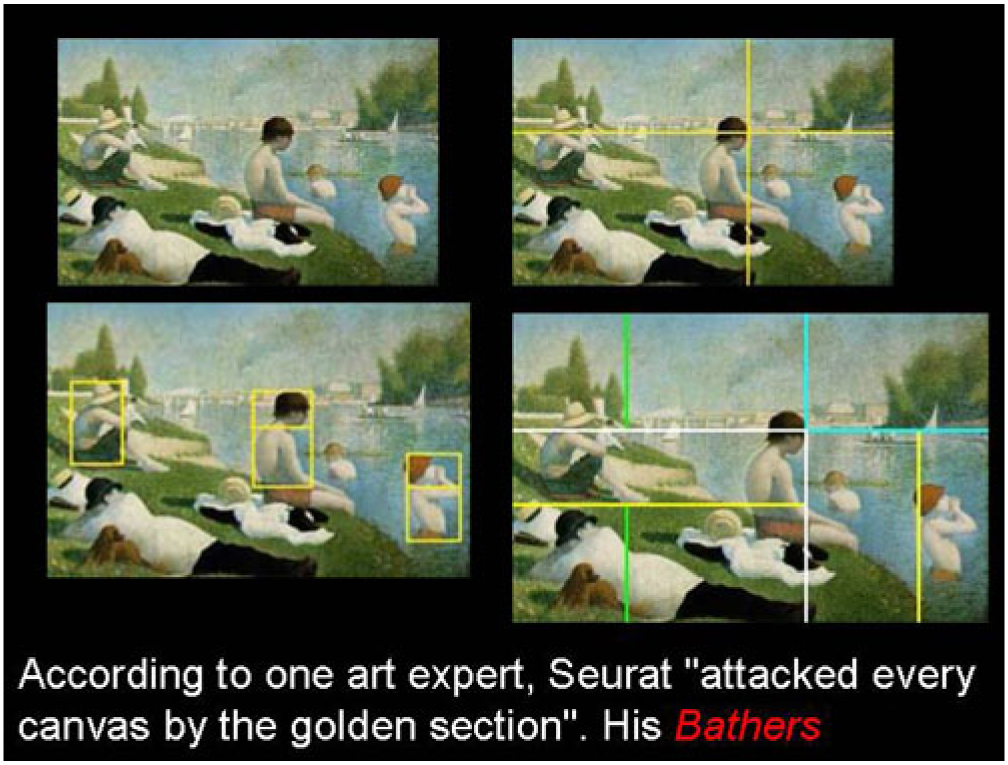
Where there is no mathematics there is no Art.

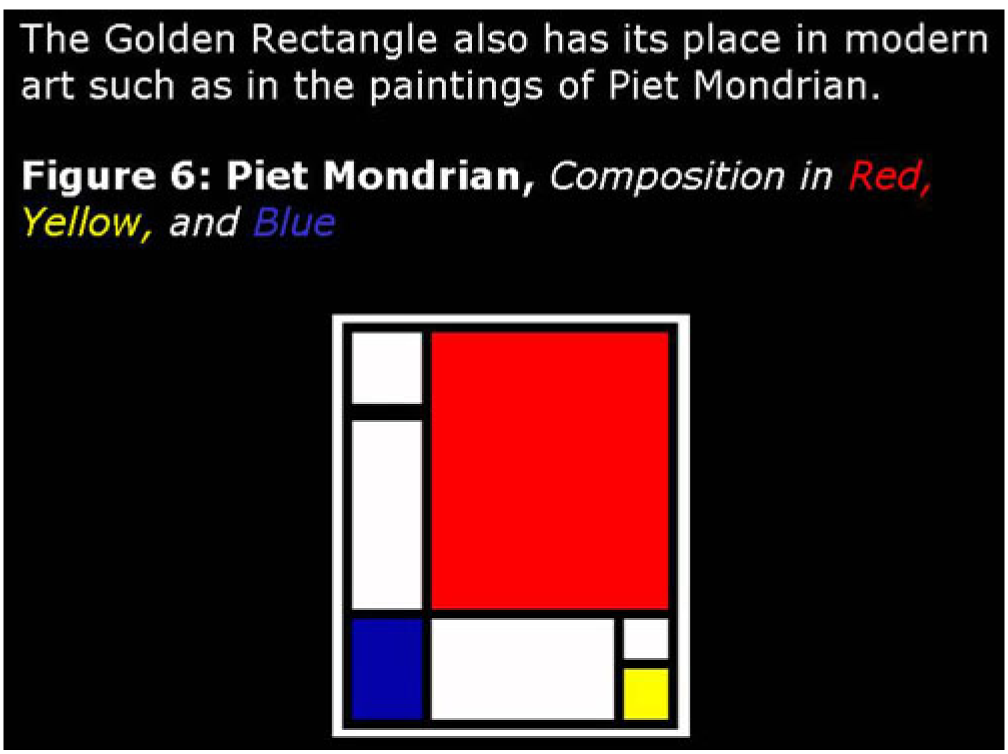
Luca Pacioli, author of *Divina pro* ***por*** *tsionay* is the central figure in this painting (by Jacopo de Barbari, 1495). Perhaps no other work so epitomizes the deep Renaissance connection between art and mathematics. Pacioli (a Franciscan friar, shown in his robes) stands at a table filled with geometrical tools (slate, chalk, compass, dodecahedron model, etc.), illustrating a theorem from Euclid, while examining a beautiful glass rombicube octahedron rhombicuboctahedron half-filled with water. Every aspect of the picture has been composed meaningfully and mathematically. Art historians have analyzed it at length, and some believe that the figure to his right is the famous artist and mathematician Albrecht Durer.

Evidence of Pacioli's work is seen in the art of Albrecht Durer, whose art showed the influence of the mathematical theory of proportion which he spent a tremendous amount of time studying. This is apparent in the woodcuts Life of the Virgin.



More recently French impressionist George Seurat's paintings contain numerous golden sections in them and many of Mondrian's paintings are just a collection of golden rectangles. Because golden rectangles are most appealing to the eye making the works more beautiful and correctly proportioned.

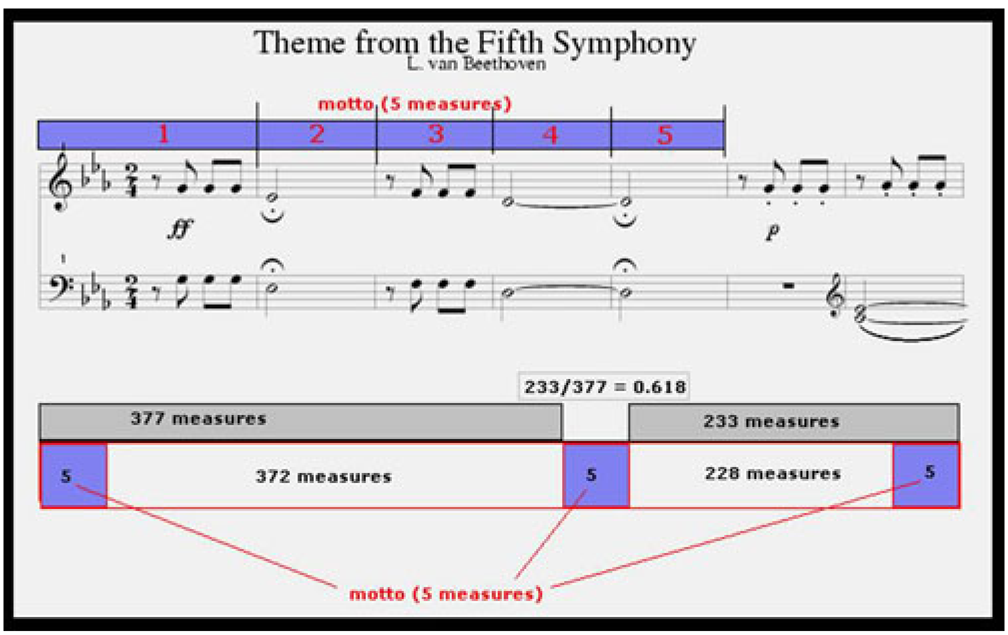




The dimensions of Salvador Dali's "Sacrament of the Last Supper" are in the Golden Ration and seen floating above the table is a dodecahedron with each side a pentagon, creating parts of the Golden Ratio.

Artists were not alone in their use of the golden ratio.

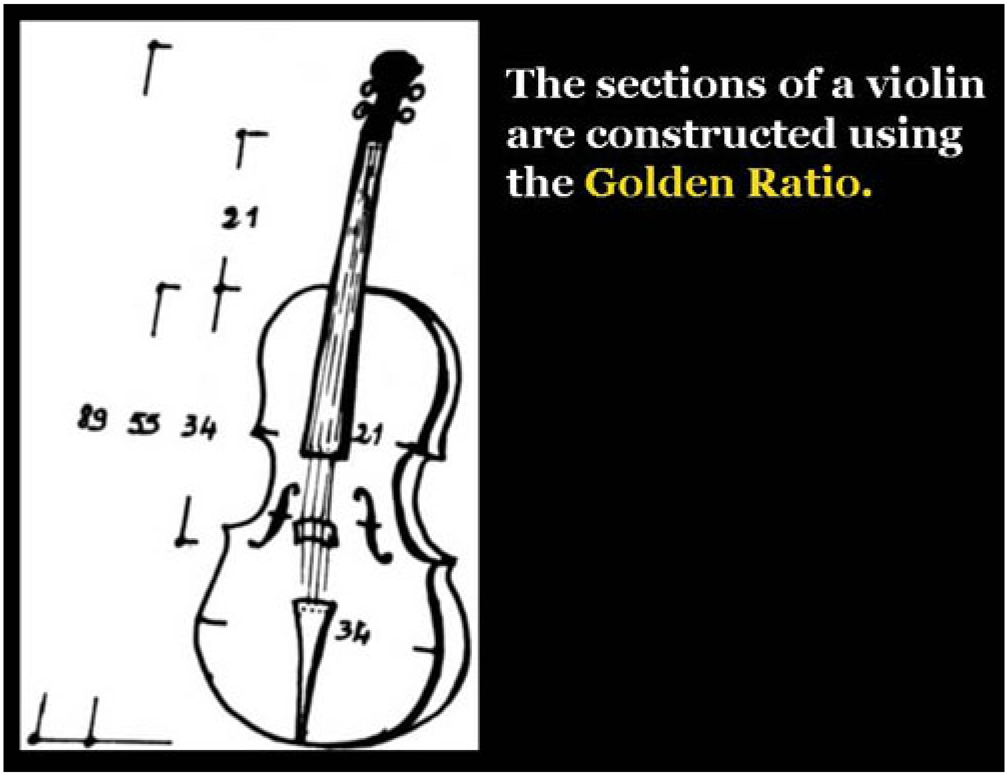
Famous musicians have also used the Golden Ratio because of its inexplicable appeal. In examining Beethoven's Fifth Symphony, for example, Derek Haylock "finds that the famous opening 'motto' appears not only in the first and last bars (bar 601 before the Coda) but also exactly at the golden mean point.



Other composers such as, Back, Mozart, and Bartok also incorporated the Fibonacci numbers and Golden Ratio into their works, either consciously or unconsciously. A report on Mozart's sonatas has revealed that they divide exactly at the golden section in almost all cases. The music that you heard as you entered the Garvin was Mozart's Piano Sonata no. 1 in C Major. The sonata-form movement was conceived in two parts: the Exposition in which the musical theme is introduced, and the Development and Recapitulation in which the theme is developed and revisited. In the piece you heard, the former consists of 38 measure while the later consists of 62 measure, which is the ratio of the golden ratio. The golden ratio is used to generate rhythmic changes or to develop a melody line. Bartok's 4th String Quartet Music for Strings, Percussion and Celesta is divided into sections of 55 and 34 bars. The 55 bar section is divided into 34 and 21 bars, these are further subdivided into Fibonacci numbers... Each division marks a change in atmosphere: the climax is situated at bar 55 of 89.

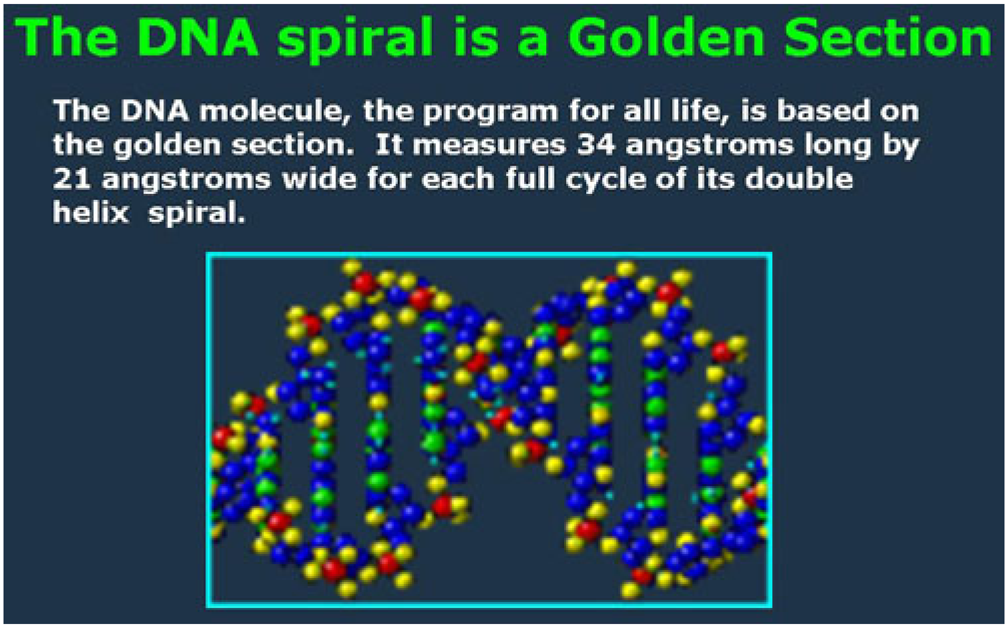
An octave of the chromatic scale has 13 notes. On a piano, 8 of these keys are white and 5 are black. The black keys of the pentatonic scale are grouped into sets of 2 and 3 keys.

Stadivari was aware of the golden section and used it to place the f-holes in his famous violins. Baginsky's method of constructing violins is also based on golden sections.

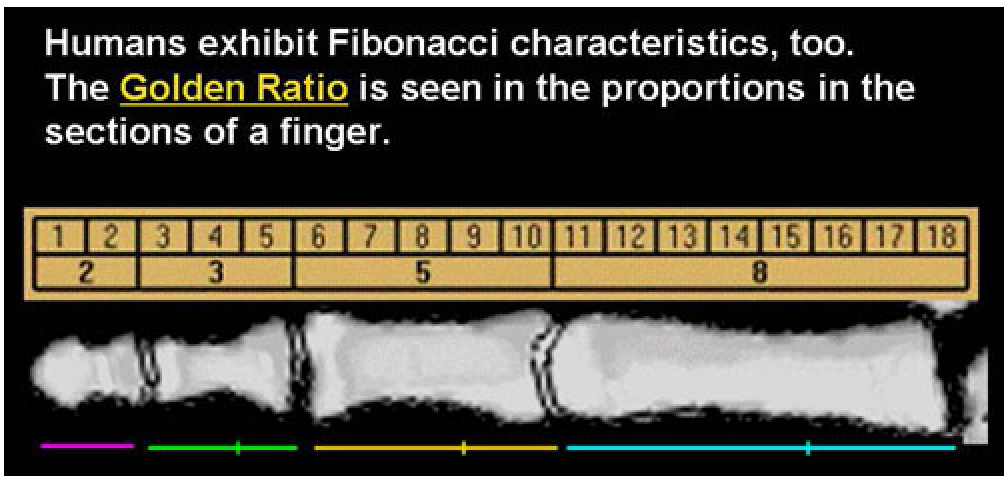


Musicians, painters, and architects all control their output, and therefore could incorporate the Golden Ratio consciously.

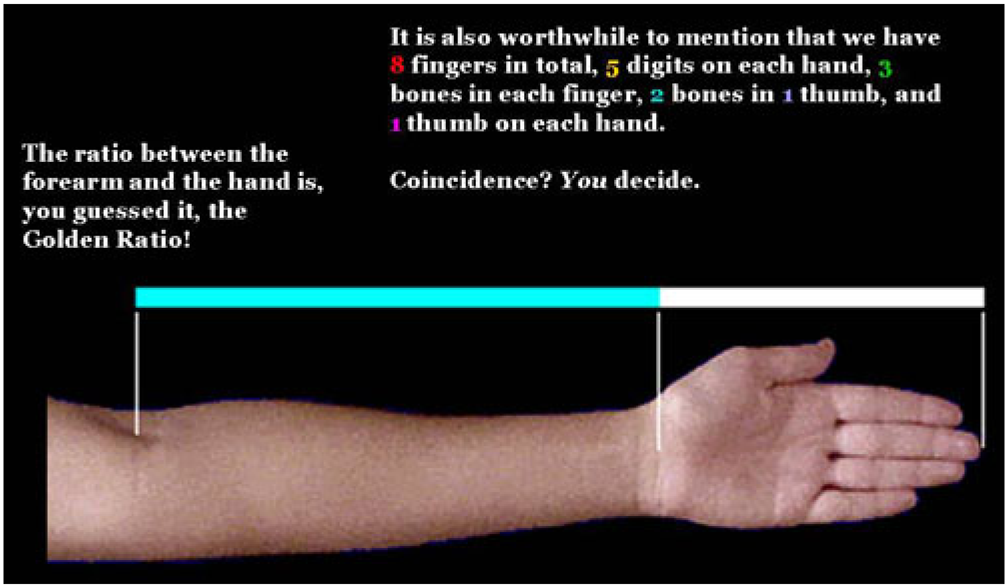
Nature, on the other hand, seems left to chance. It is most striking, then, that the Golden Ratio also appears in nature, where humans could not have forced mathematics. The golden ratio appears in nature from angel fish, to penguins and from ants to tigers, but the human body abounds with examples of the golden ratio. The DNA molecule is based on the Golden section measuring 34 angstroms long by 21 anstroms wide. The ideal blood pressure is 120/75 which is equal to the golden ratio.



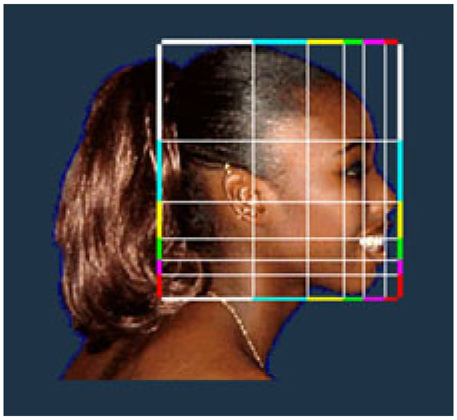
Each section of your index finger is in the proportion of the golden ratio.



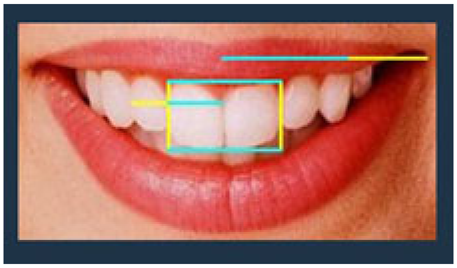
Your hand creates a golden ratio in relation with your forearm. The distance from your finger to your shoulder and your finger to your elbow is in the proportion of the golden ratio.



The head forms a golden rectangle with the eyes at its midpoint. The ratio of the avereage width of a human mouth to a human nose is 1.61. Appropriately enough, this ratio is commonly used in facial reconstructive surgery when a sense of balance is desired.



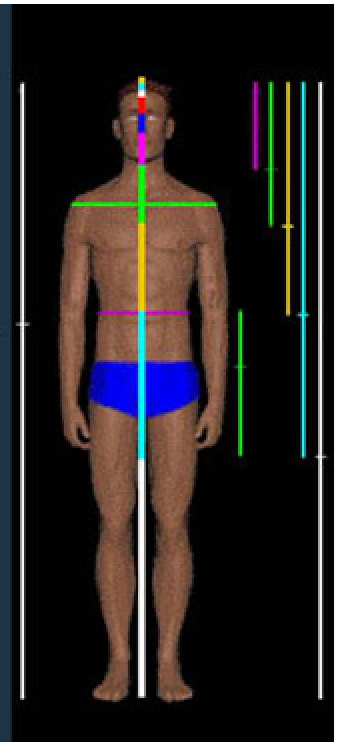
Our smile. The front two incisor teeth form a golden rectangle.



We could go on for a good while but I will share with you my favorite golden ratio. If I measure the distance from the ground to my belly button it's 45.5 inches. If I multiply this value by phi or approximately 1.61 I get my height of 74 inches.

The Divine Proportion in the Body

The white line is the body's height.
The blue line, a golden section of the white line, defines the distance from the head to the finger tips
The yellow line, a golden section of the blue line, defines the distance from the head to the navel and the elbows.
The green line, a golden section of the yellow line, defines the distance from the head to the pectorals and inside top of the arms, the width of the shoulders, the length of the forearm and the shin bone.
The magenta line, a golden section of the green line, defines the distance from the head to the base of the skull and the width of the abdomen. The sectioned portions of the magenta line determine the position of the nose and the hairline.
Although not shown, the golden section of the magenta line (also the short section of the green line) defines the width of the head and half the width of the chest and the hips.



Another art form would be in literature, well almost literature, the limerick. Most limericks aspire to rhythm or pattern of unstressed (u) and stressed (S) syllables. The combination of uS and uuS are called metrical feet. I have composed an original limerick for you today and we'll analyze it.

The Limerick

There once were two men who loved Phi
We know one was Fibonacci
His book was a hit
About a rabbit
The other was named Da Vinci

Most limericks aspire to rhythm or pattern of unstressed (u) and Stressed (S) syllables. The combination **uS** and **uuS** are called metrical feet. See is you can find the occurrence of the Fibonacci Numbers in the underlying structure of the limerick.

		u	S	Tot
There **once** were two **men** who loved **Phi**	uSuuSuuS :	5	3	8
We **know** one was **Fibonacci**	uSuuSuuS :	5	3	8
His **book** was a **hit**	uSuuS :	3	2	5
About a rab**bit**	uSuuS :	3	2	5
The **other** was **named** Da Vinci	uSuuSuuS :	5	3	8
	Total :	21	13	34

I hope you have enjoyed this most amazing and elegant number with me.

I opened with Donald Duck in Mathmagicland because it was what I have always tried to do in my 35 years of teaching. What I have always tried to do is teach a little math and have a little fun. So, we come to an end and I do hope you have learned a little bit of math and had a little bit of fun. I did. I had a lot of fun. This is what I like to do and today I sure had a good time doing it. I thank you all so much for coming and sharing this most precious day with me.

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Internet Resources

Eric Weisstein's Mathworld, mathworld.wolfram.com:

Golden Ratio, Fibonacci Number, Logarithmic Spiral

Sloane's Integer Sequence Database: Fibonacci, Lucas

Mario Livio's new book from the Seminary Co-op

Ron Knott's (Surrey University) expansive website

Journal: The Fibonacci Quarterly

Smith College Mathematics department: Phyllotaxis

UC Irvine Geometry Junkyard: Penrose Tilings

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Banana cross section - http://www.goldennumber.net/plants/
Apple cross section - http://www.goldennumber.net/plants/
Credit card - http://www.goldennumber.net/credit-cards/
Parthenon - http://www.goldennumber.net/architecture/
Notre Dame - http://www.goldennumber.net/architecture/
Seurat painting - http://www.goldennumber.net/art-composition-design/
Teeth - http://www.goldennumber.net/face/
DNA spiral - http://www.goldennumber.net/dna/
Human finger - http://www.goldennumber.net/human-hand-foot/
Human arm - http://www.goldennumber.net/human-hand-foot/
Human face side view - http://www.goldennumber.net/face/
Human body - http://www.goldennumber.net/human-body/